On the Groupings and General Behaviour of Solid Particles under the Influence of Air Vibrations in Tubes

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XII. *On the Groupings and General Behaviour of Solid Particles under the Influence of Air Vibrations in Tubes.*

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(PLATES 21–26.)

1. *Introduction.*

IN 1866 KUNDT showed, by a method described in every text-book of Sound, that if stationary waves are set up in the air in a long tube containing dust, then the wave-length is indicated by little collections of dust which form at the nodes. In addition to these dust heaps at the nodes there appear, when the air is thrown into strong vibration, a series of ridges or striations of dust, lying on the bottom of the tube at right angles to the axis. Photographs illustrating these ridges are appended to this paper, and will receive further reference at a later stage. The production of these striations (which were, of course, observed by all the earlier experimenters not while the air was in vibration, but after the vibration had ceased) was explained in 1891 by WALTER KOENIG,* in a calculation of hydrodynamic forces, based on investigations of C. A. BJERKNES.†

KOENIG considered the case of two spheres in a vibrating fluid, and showed that if the line joining the centres of the spheres was parallel to the direction of vibration of the fluid there was repulsion between them, while if the line of centres was normal to the direction of vibration the spheres attracted one another. The force thus called into play varies inversely as the fourth power of the distance between the centres of the spheres, and directly as the square of the velocity of the fluid. It is assumed that the spheres do not share appreciably the periodic motion of the fluid—an assumption which is generally, but not always, reasonable for obstacles of the size and mass used, viz., particles of cork dust and the like. To carry out the calculation a number of

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* W. KOENIG, 'Ann. Physik,' vol. 42, p. 353 and p. 549 (1891); vol. 43, p. 43 (1891).

† See V. BJERKNES, 'Vorlesungen über hydrodynamische Fernkräfte, nach C. A. BJERKNES Theorie,' 1900–2.

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subsidiary hypotheses have to be made, of which the most significant is the neglect of the viscosity of the fluid. This assumption, which has been generally accepted, might well have thrown doubt on the validity of KOENIG'S theory, since, as a matter of fact, no striations are formed unless the amplitude of vibration exceeds a certain threshold value (see Section 2), and it is known that such a threshold value, reached as we proceed to large velocities, generally signifies that the medium has ceased to behave even approximately as a perfect fluid.

KOENIG supposed that the ridges were formed by the action of the repulsive forces between the spheres along the axis of the tube. The difficulty arises that, if only repulsive forces exist in this direction, there seems no reason why the particles should arrange themselves in regular striations, although if the striations already existed the variation of the forces with the square of the velocity might be invoked to explain the fact that the ridges are most widely spaced at the antinode. KOENIG somewhat diffidently states: "It remains an open question whether the distance of the two ribs must be regarded as the measure of the distance at which the forces are already too weak to occasion further movement of the dust particles. For repulsive forces act on both sides of any cross ridge, and for the stable position of such a wall it would be a sufficient condition that the difference of the forces should be small enough not to be effective. That the ridges are actually formed very uniformly and in regular spacing from one another follows without difficulty from this kind of joint action of forces." From this it would appear that KOENIG invokes frictional forces. Later workers, *e.g.* J. ROBINSON,[*] have expressed themselves as fully satisfied with KOENIG'S theory, and ROBINSON even states expressly: "It has been suggested that these forces are not in themselves sufficient and that some constraint must be introduced, such as, for instance, one due to viscosity. In the present paper it will be shown that no such constraint is necessary."

KOENIG[†] adduces in support of his theory experiments subsequently published by GEORG THOMAS,[‡] who investigated the forces between two spheres in vibrating air. Approximate, but not close, agreement with his theory was obtained, the agreement being best for spheres whose radius was small compared to the distance between them. On the other hand, S. H. COOK,[§] working with two glass spheres in a vibrating gas, found that when the line of centres was parallel to the direction of vibration, and the distance apart of the spheres was less than half the diameter, the suspended sphere approached the fixed sphere and moved around until the line of centres was across the direction of vibration—observations which THOMAS,[||] in an extended experimental investigation, considered that he had refuted: "Die Beobachtungen haben dargetan,

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[*] 'Proc. Lond. Phys. Soc.,' vol. 25, p. 256 (1913).

[†] 'Phys. Z.,' vol. 12, p. 991 (1911).

[‡] 'Ann. Physik,' (4), vol. 42, p. 1079 (1913).

[§] 'Phil. Mag.,' (6), vol. 3, p. 471 (1902) and (6), vol. 6, p. 424 (1903).

[||] *Loc. cit.*, p. 1087.

dass die Kräfte stets die Richtung hatten, die sich aus der Theorie für reibungslose Flüssigkeiten ergab." THOMAS states categorically that repulsion for the equatorial position and attraction for the axial direction never takes place, even when the spheres almost touch. In a recent paper IRONS* has recorded measurements of the distance between striæ which, as far as their irregularity leaves them any significance, fail to support ROBINSON.

It it clear, then, that in the first place the mechanism of the formation of the ridges, although the subject of investigations from 1866 to the present day, has never been satisfactorily explained, and that in the second place there has been considerable conflict as to the force existing between two spheres in a vibrating gas, and, in particular, contradictory experimental evidence as to the *sense* of the force between close spheres.

In another place† experiments have been described which proved the existence of a vortex motion in the air round an obstacle in a vibrating gas, providing that the amplitude of the vibration exceeds certain limits governed by the principle of dynamical similarity. Under the conditions ordinarily prevailing in a sounding tube, and in particular the conditions of THOMAS's and COOK's experiments, this vortex motion takes place. Neglect of this fact is the cause of the obscurity and contradiction which has been connected with the subject. The conditions of vortex-free motion postulated in KOENIG's theory do not hold when ridges are formed ; they can be made to prevail by reducing the intensity of the sound, but precisely when these conditions hold the ridges, which KOENIG's theory is constructed to explain, do not form at all.

In the following pages it is shown how vortex motion, combined with the general circulation between antinode and node described in the paper just cited,† can be made to account for the dust figures hitherto observed, and new features of the behaviour of dust particles in a vibrating gas are described and explained.

2. *The Older Dust Phenomena.*

The observations hitherto published on the dust groupings in a sounding tube suffer first of all from a lack of system, and secondly, from the fact that the observations have all been made with intermittent sounds produced by stroking a rod. Not only does this procedure mean that the observations and measurements are nearly all carried out after the sound has ceased, but also the conditions which produce the groupings are never maintained long enough for it to be certain that the final stable arrangement, corresponding to the particular vibration of the air, has been reached. Again, the intensity of the sound so produced grows to a maximum and dies down again within the short duration of the stroking of the rod, so that, even in a rough and qualitative way, it is hard to estimate the intensity to which the grouping corresponds.

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* E. J. IRONS, 'Phil. Mag.,' (7), vol. 7, p. 523 (1929).

† "On the Circulations caused by the Vibrations of Air in a Tube" (referred to in future as "Circulations"). 'Proc. Roy. Soc.,' A, vol. 134, p. 445 (1931).

The range of frequency which has been employed has also in general been limited by the necessity of employing rods of reasonable length, which, in consequence of the high velocity of sound in solids, means that the frequency is in the neighbourhood of 3000∼, *e.g.*, *circa* 3900 in KUNDT's original experiments, *circa* 2800 (apparently) for DVORAK's* experiments, *circa* 3200 for those of ROBINSON† and *circa* 2500 for those of IRONS.‡

It is of fundamental importance to be able to observe the phenomena in a steady state, while the air is vibrating with constant amplitude at any given place; to be able to vary the frequency and to be able to vary the amplitude at will, all of which can be done with the apparatus described in the paper on "Circulations,"§ Section 2. The vibrations are maintained by a valve-driven diaphragm. It is also an advantage to be able to observe the wave form, which can be done with the cathode ray oscillograph, as there described. There is an additional feature to be observed, namely, whether the particle shares, to a greater or less extent, the oscillatory motion of the air, or whether, except for slow non-periodic drift, it is at rest, which behaviour depends upon the amplitude and frequency of the air vibration and the size and mass of the particles which make up the dust. Both extreme cases, the one where the particle is fully at rest, and the other where it moves with the full periodic amplitude of the air vibration (the case of the smoke particles described by the writer elsewhere§), have been realised, the case of most frequent occurrence being the intermediate one where the particle executes a vibration, but not with the full amplitude of the air vibration. This case leads to more complicated phenomena.

For most of the following experiments on ridge formation with dust, fine cork dust particles of approximately uniform size were used. The dust was graded by sieving, but always contained, in addition to the approximately uniform particles, some of extremely small size. A photograph of a typical sample of the dust, magnified 50 times, is given in fig. 5, Plate 21. The minor problems connected with the nature of the particle, whether cork, lycopodium, silicic acid, iron filings, sunflower pith dust or egg powder, to quote some of the substances used by other experimenters, have not been considered at length in this paper.

If a line of cork powder be placed in a well-dried tube and the diaphragm be excited by a range of continuously varying frequencies which extended, in these experiments, from 2285 to 100 ∼, at different discrete frequencies the cork dust forms up into well-marked ridges, groups of ridges being separated by patches of quiescent dust, indicating the nodes. At the higher frequencies the walls of dust are extremely sharp, being but one particle thick and quite straight, the top of the wall being approximately horizontal (slightly convex upwards) and the bottom, of course,

* 'Ann. Physik,' vol. 157, p. 42 (1876).

† 'Phil. Mag.' (6), vol. 19, p. 476 (1910).

‡ 'Phil. Mag.' (7), vol. 7, p. 523 (1929).

§ *Loc. cit.*, p. 448.

circular in conformity with the tube, so that the walls taper away at the edges into single particles. Fig. 6, Plate 21, shows the walls as seen obliquely from above; the individual particles can be plainly seen, well separated, in the ridges which are sharply in focus. Another general feature of the ridges is the tendency to form a less defined wall, containing fewer particles and of less height, between each main ridge. It is clearly shown in figs. 7 and 9, Plate 21, which are photographs of ridge systems as seen from vertically above, taken, like fig. 6, while the note was sounding. This feature has been previously noticed,* but no explanation offered. The minor ridge is important for the interpretation of the general phenomena offered in this paper and will receive further reference in Section 5. Fig. 8 (Plate 21) shows a system of ridges obtained under the same conditions as those of fig. 7, but with the minor, intermediate ridges suppressed. This can be done by tapping the tube sharply. The reason for this is given later.

Before describing in further detail the essential features of ridge formation, reference will be made to the chief earlier observations. The observations of KUNDT† were as follows. Using lycopodium and an exciting rod of fixed length and material, he varied the length of the tube containing the air, which corresponds to the variation of frequency with a fixed tube length in the present experiment, and found that when the length was such that a node was formed just in front of the driving disc, the dust collected in heaps at the nodes, with no other dust figures in the tube. Each heap was prolonged at the sides in a streak, normal to the generators of the tube, which passed some way up the sides of the tube—the heaps "ziehen sich noch ringförmig an den Seitenwänden der Röhre in die Höhe." When the length was such that an antinode was formed near the driving disc, a different figure was formed, with what KUNDT calls "holes" at the nodes—a type of figure which I have called an "eye" in this paper. These "holes" are illustrated in Plate 24; one is seen especially well in figs. 31 and 32.

When sand was used, or fine iron filings (in other words, particles of dense material), and the length was such that a node was formed at the driving disc, a series of sharp ridges was formed between each pair of nodes, the nodes themselves being distinguished by dust heaps, while, with such particles, if the length of the air column was such that an antinode was near the driving disc, there was no movement of the particles. These represent the essential observations up to the present date, the observations of DVORAK, IRONS and others not having added anything essentially new as regards the dust patterns.

All these features have now been traced to differences of intensity consequent on, and in conjunction with, differences of frequency, and can be obtained at will by varying *either* frequency *or* intensity in a way to be described.

The highest frequency for which movement of dust was studied in the present investigation was in the neighbourhood of 2300 $\sim$, *e.g.*, 2285, with a particular tube 163 cm. in length. At such frequencies sharp ridges with straight, horizontal tops

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* *E.g.*, IRONS, *loc. cit.*

† 'Ann. Physik,' vol. 127, p. 497 (1866); vol. 128, p. 337 (1866).

are formed, which are longer at the antinodes, and decrease symmetrically in size on either side, the final ridge being some distance from the node, as clearly shown in figs. 7 and 8, Plate 21, obtained with frequency 1240. Diminishing the intensity at this frequency causes the whole ridge system to shrink away from the nodes, each individual ridge becoming less high and, in consequence, decreasing in length, while the ridges nearest to the nodes disappear. Further decrease in intensity causes the ridges to collapse altogether into little heaps on the bottom of the tube, there being no movement of the dust, although the sound is still of some intensity.

The frequencies at which these high-frequency ridges appear are fairly sharply defined, there being little movement of dust for frequencies a few periods above or below the critical one. As the frequency is steadily lowered a series of critical values is obtained, at each of which sharp ridges are formed, which are affected by changes of intensity in the way just described.

3. *Phenomena at Lower Frequencies : The Antinodal Disc.*

The behaviour just described characterises a range of high frequencies. When, diminishing the frequency, a certain value is reached, a new type of behaviour is observed, which is characteristic of a range of medium frequencies. The most striking feature is the formation of a sharp disc not confined to the bottom of the tube, but extending across the whole section, *at the antinode.** This disc must not be confused with what is sometimes termed the "ring" which forms *at the nodes*, namely, the extension of a nodal heap of dust into a sharp ridge running some way up the sides of the tube as noticed by KUNDT. A picture of such a disc is given in figs. 11, 12 and 13, Plate 22. Fig. 11 shows an oblique view ; fig. 12, a disc as seen from the side ; fig. 13, as seen from above. Fig. 14 shows five antinodal discs in a sounding tube. The disc is extraordinarily sharp, being in some cases much less than 1 mm. thick, and very steady when conditions are carefully controlled.† The mode of formation of these discs is further considered in Section 8.

The disc forms up at any one of a small range of frequencies in the neighbourhood of a certain critical frequency. By adjusting the intensity the disc can be made very sharp, the optimum intensity being only slightly in excess of the minimum required to form a disc at all. When the disc is formed ridges on the bottom of the tube are also present and steady, but they are somewhat blurred in the neighbourhood of the disc itself, as can be seen in fig. 13, owing to the large amplitude of the air vibrations at the antinodes which is shared, to a greater or less extent, by all the particles in the ridges. By reducing the intensity, keeping the frequency fixed, the ridges can be made quite

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* First announced in 'Nature,' Nov. 9, 1929. Attention is called to the existence of this disc by E. Hutchisson and F. B. Morgan, 'Phys. Rev.,' vol. 37, p. 1155 (1931).

† Mr. D. H. SMITH and I are using these discs to measure sound velocity, and have already found it possible to obtain with them values consistent to 1 part in 2000.

sharp, but the intensity corresponding to this is below the minimum necessary for the formation of a disc.

Keeping to the full intensity of the oscillator, two narrow ranges of frequencies are found, one on either side of the antinodal disc frequency, at which sharp and steady ridge systems appear. For brevity, the conditions at which these ridges appear will often be called " ridges high " and " ridges low," according as the higher or lower frequency is in question. Such a group of three frequencies—ridges high, antinodal and ridges low—appears at regular intervals as the frequency is lowered further and further.

At a frequency slightly different from the antinodal frequency the ridges slowly travel out from the antinode towards the node and ultimately (*i.e.*, after a duration of a few or several minutes), if the vibration is maintained, all the dust is cleared to the neighbourhood of the nodes. The frequency at which this phenomenon takes place is that at which the response of the tube is most vigorous. The mode of progression is peculiar. The particles at the outer edges of each ridge, where the ridge is only a particle or two high, move along from ridge to ridge and finally a very small ridge is formed near the node which, partaking of the general motion of the ridges, moves slightly towards the node and then collapses on the floor of the tube. Successive small ridges are so formed, move towards the node, and collapse at the same place, building up a thick line of dust near the node, but not *at* the node—an important point. In this process the central ridges gradually become entirely depleted of dust so that two systems are formed, as shown in fig. 30, Plate 24, which depicts conditions while the clearance is in progress. The process continues, the last ridge on the antinodal side being robbed of its dust from the edges and vanishing, to leave its neighbour as the last ridge, which vanishes in its turn, until all the dust has passed into a single wall near the nodes, which, in consequence of the large amount of dust deposited, and of the fact that it is mainly deposited as short ridges, has a heap of dust in the middle, as shown in fig. 31, Plate 24.

This wall should be regarded as a ridge deposited at a position where the amplitude just ceases to be sufficient to maintain a ridge. Further explanation is given in Section 7. Two of these lines, one on either side of the node, as shown in fig. 31, Plate 24, form one of the eyes described by KUNDT. If the vibration is very vigorous the two lines are formed so close together that they coalesce as one broad line at the node. This is KUNDT's observation for light powder when the tube length was such that a node was formed near the driving disc.

Slight modifications of the phenomena depend on the amount of dust present. If there is plenty of dust the ridges are high and present a peculiar wavy appearance near the antinode, a kind of flickering in and out, of an S shape, but only in the case of the central ridges. This is explained in Section 8.

All the phenomena just described take place in the range of middle frequencies. As the frequency is lowered still further the phenomena are again modified. Critical frequencies still occur at which an antinodal disc is formed, but the disc is very wide, owing to the large amplitude of the air vibrations, which, at low frequencies, entrain the dust particles. The ridges likewise appear extremely wide, and fluffy in outline, for the same reason. An example is given in fig. 10, Plate 21, obtained with a frequency 120 ∼. The particles of cork thrown into the air by the motion take part in the vibratory motion of the air, and appear drawn out into lines, which at low frequencies, with the cork particles used, indicate very nearly the full amplitude. The ridges also extend in height so as to occupy nearly the whole diameter of the tube at the antinode.

By lowering the intensity at low frequencies the ridges can be made much sharper, but never attain the sharpness which they have at high frequencies, collapsing first. The travelling out to form a nodal eye can be obtained by slightly varying the frequency, but ridge systems characteristic of the two frequencies called ridges high and ridges low do not appear.

At the lowest frequencies (*e.g.*, *circa* 75 ∼) and large intensity the appearance is very remarkable, the ridges in the neighbourhood of the antinode becoming over 3 cm. thick, owing to the particles taking up in full the very large amplitude of the air motion. They also extend right across the tube, forming five or six complete walls, with ridges of less height to either side.

In the course of the experiments made to elucidate the causes of the various phenomena many runs of quantitative measurements of the frequencies associated with them were made. A series of such measurements is given in Table I. Ridges high (R.H.) denotes the formation of ridges at the higher of a pair of associated frequencies at which sharp ridges appear; ridges low (R.L.) the formation of sharp ridges at the lower of the pair of frequencies; clearance (C.) denotes the movement of the dust to form a nodal eye, leaving the antinodal region bare; antinodal disc (A.N.D.) denotes the appearance shown in fig. 11, Plate 22. n is a whole number, giving the resonant frequency as a multiple of the fundamental frequency. The frequency given under "Resonant Frequency" is, for the high series, *i.e.*, frequency 2285 to 1324, the mean of the ridges low and ridges high frequencies; for the medium series, *i.e.*, frequency 1244 to 719, it is the antinodal disc frequency, or, if the phenomenon appears, the clearance frequency; for the low series, *i.e.*, 653 to 200, it is the clearance frequency.

The reason for this is that, as will be shown later, the appearance of the various phenomena is mainly governed by the intensity of the air vibration; clearance, antinodal disc and ridges corresponding to intensities in diminishing order. Hence, if clearance takes place, the frequency is that at which the air vibration is most vigorous. If an antinodal disc is formed, but no clearance takes place, the formation of the disc corresponds to the most vigorous vibration. If the ridges are formed at two near frequencies the appropriate energies must correspond approximately to frequencies displaced somewhat to either side of the resonant frequency, the intensity at resonant frequency being too great for sharp ridges but not great enough to form an antinodal disc. It is of importance to note that no particular phenomenon, as such, corresponds to resonant conditions. The best way of finding the true resonant frequency is to measure the energy fed into the oscillator unit, as will be described in a subsequent communication.

Table I.—Frequencies at which the various dust phenomena take place. Tube length = 163 cm., diameter = 3·5 cm., temperature = 20·1° C.

Frequency.	Phenomenon.	Resonant Frequency.	*n*.	102·58 *n*.	Frequency.	Phenomenon.	Resonant Frequency.	*n*.	102·58 *n*.
2285 2220	R.H. R.L.	2253	22	2257	1152 1136 1124	R.H. A.N.D. R.L.	1136	11	1128
2187 2128	R.H. R.L.	2157	21	2154	1054 1034	R.H. A.N.D.	1034	10	1026
2055 2033	R.H. R.L.	2044	20	2052	943 926 919 894(?)	R.H. A.N.D. C. R.L.	919	9	923
1948 1925	R.H. R.L.	1937	19	1949	857(?) 829 820	R.H. A.N.D. C.	820	8	821
1850 1830	R.H. R.L.	1840	18	1846	763 731 719	R.H. A.N.D. C.	719	7	718
1750 1730	R.H. R.L.	1740	17	1744	653 619	A.N.D. C.	619	6	615
1655 1635	R.H. R.L.	1645	16	1641	521 512	A.N.D. C.	512	5	513
1536 1516	R.H. R.L.	1526	15	1539	422 413	A.N.D. C.	413	4	410
1442 1426	R.H. R.L.	1434	14	1436	325 315	A.N.D. C.	315	3	308
1344 1324	R.H. R.L.	1334	13	1334	200	C.	*circa* 200	2	205
1244 1234 1218	R.H. A.N.D. R.L.	1234	12	1231					

The uncertainty in the neighbourhood of frequency 870 ~ is caused by a resonance of the diaphragm.

This table is inserted to give a general view of the way in which the various phenomena appear. No importance attaches to the exact frequencies at which ridges are formed. Intensity being the true governing factor, the frequency corresponding to any phenomenon can be changed simply by changing the energy supplied to the driving diaphragm, *i.e.*, by the insertion of resistance in the output circuit.

On the general scheme which has been devised to cover the phenomena it would be expected that when clearance takes place at a given (resonant) frequency, slightly different frequencies, at which the intensity is sufficient to produce an antinodal disc, would be found close to, and on either side of, the resonant frequency. It has not been found possible, however, to detect two antinodal disc frequencies in the way that two ridge frequencies are found. In the table the antinodal disc frequency is always slightly higher than the clearance frequency. With other tubes an antinodal disc frequency slightly lower than the clearance frequency has sometimes been detected. In one particular case on a higher range of frequencies the antinodal disc frequency was lower than the clearance frequency; over a lower range things were reversed, corresponding to the case represented in the table. No particular importance is attached to this phenomenon, which is attributed to minor influences affecting stability. It should be noted that when clearance is taking place an antinodal disc is temporarily formed which persists until eventually all the dust is removed from the antinodal position to the nodes.

4. *The Formation of Ridges: a Crucial Experiment.*

The conditions for the formation of sharp ridges have been shown to be sufficiently high frequency and an amplitude within a certain restricted range, too high an amplitude leading to a general motion of the particles which makes the ridges blurred, and too low an amplitude not being able to produce the phenomenon at all. In illustration of this two general observations may be cited. When the amplitude is large, a few sharp ridges are formed for a limited region near the nodes, although the ridges at the antinodes are blurred, which demonstrates the unfavourable effect of too large an amplitude. The fact that the ridges never extend right up to the nodes shows that a certain minimum amplitude of motion is necessary.

The amplitude has been cited as the criterion, since it is open to direct observation, but general considerations of the hydrodynamics of viscous motion would suggest that it is the velocity of the air past the particles, possibly the acceleration, that constitutes the criterion. The principles of dynamical similarity afford some information on this point.

Suppose that the force F between two spheres of diameter h, in a vibrating medium of density ρ and kinematic viscosity ν, depends upon the velocity v and acceleration a of the medium, as well as upon h, ρ, ν and the distance l between the spheres, in a way given by

$$\mathrm{F} = \rho^{x}\, v^{y}\, \nu^{z}\, a^{r}\, h^{p}\, l^{q}.$$

Equating powers of M, L and T, we have

$$\begin{aligned} x &= 1 \\ -3x + y + 2z + r + (p + q) &= 1 \\ y + z + 2r &= 2. \end{aligned}$$

Applying the principle of dynamical similarity in the ordinary way, we obtain

$$\mathrm{F} = \rho v^2 l^2 \,.\, f\left(\frac{\nu}{vl}, \frac{al}{v^2}, \frac{h}{l}\right)$$

showing the dependence of the force on velocity and acceleration.

If we assume a simple harmonic vibration of the medium,

$$s = \mathrm{A} \cos 2\pi\,(nt + \varepsilon)$$

then

$$v = 2\pi ns = n\mathrm{A}$$

$$a = 2\pi n^2 s = n^2\mathrm{A} \quad \text{dimensionally,}$$

so that

$$\mathrm{F} = \rho n^2 \mathrm{A}^2 l^2 \,.\, f\left(\frac{\nu}{n\mathrm{A}l}, \frac{l}{\mathrm{A}}, \frac{h}{l}\right).$$

Koenig's calculation neglected ν, and assumed, as above, a simple harmonic motion. He found an expression dimensionally equivalent to $\mathrm{F} = \rho n^2\,\mathrm{A}^2\,l^2\,(h/l)^6$, in which f has a particularly simple form. The general expression shows, as might be anticipated, that it is sufficient, with a given medium and given bodies, to fix n, A and l to determine the force.

Koenig's theory supposes that the motion of the air is vortex-free. It seemed to the writer that since, as has been pointed out, a minimum velocity of the air is necessary for the formation ridges, and since this minimum is fairly sharply defined, it was far more likely that the inception of ridge formation corresponded to that breakdown of vortex-free motion which always occurs in the case of viscous fluids when the velocity exceeds a certain minimum, as is well illustrated by, *e.g.*, the formation of vortices between two concentric cylinders, fully discussed by G. I. Taylor.* The experiments described in the paper on " Circulations " have shown that, similarly, vortex motion sets in round a particle in a vibrating viscous fluid when certain conditions of amplitude and frequency are fulfilled.

Careful observation of ridge formation in a dust tube reveals the fact that quite often, when there is but little dust, as, for instance, in the neighbourhood of a node, an isolated system of only two or three ridges forms. This alone suffices to make it exceedingly unlikely that ridges are formed in consequence of the forces calculated by Koenig, namely a repulsion in the line of vibration, falling off as the inverse fourth power of the distance. Koenig himself, as has been pointed out, was not unaware of difficulties in the way of his explanation, while Robinson, who found the explanation sufficient, demanded as a necessary condition that there should be a whole system of ridges to

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* G. I. Taylor, 'Phil. Trans.,' A, vol. 223, p. 289 (1923). J. W. Lewis, 'Proc. Roy. Soc.,' A, vol. 117, p. 388 (1928).

produce the balancing forces which he required. If two ridges only can form in such a way that the distance between them has a fixed value which is re-established if the ridges are disturbed in any way, KOENIG's theory must be fundamentally unable to explain ridge formation, for the only way in which a repulsion can explain two isolated ridges is by the dust particles being driven apart into two groups, the distance increasing until the repulsive force is too feeble to overcome frictional forces. If this were the mechanism, the distance between the two ridges would not be either sharp or stable as it actually is, nor would the ridges themselves be sharp. It is also impossible to explain on this theory another fact of observation, namely, that if a group of two or three ridges is formed with a high intensity of vibration, and the intensity then be diminished, the distance between the ridges diminishes.

To establish beyond question the inadequacy of KOENIG's theory, and to throw further light on the mechanism of ridge formation, experiments were carried out with small light spheres, about 2 mm. in diameter. They were prepared from pith by cutting with a sharp razor by hand. A number of these spheres form good ridges in vibrating air, as shown in fig. 25, Plate 23, the spheres not only forming up in lines normal to the axis of the tube, but mounting upon one another in walls two or three spheres high.

A single row of spheres was then cemented to the wall of the tube in a line normal to the axis and extending over a sector subtending an angle of about 80° at the centre of the tube. Actually there were twelve spheres in a row. Some free spheres were introduced into the tube, and the air set in vibration with the full intensity of the set and a frequency 387 $\sim$ for the case shown in the photograph. A single row of nine free spheres formed itself parallel to the fixed row as shown in fig. 20, Plate 23, the fixed row of spheres being on the left. The number of spheres was, of course, decided by the number in the neighbourhood of the fixed row when the vibration started. If the free spheres were arranged, by shaking the tube, so as to lie near to the fixed row when the air was at rest, on excitation they were repelled to the stable distance already found; if, on the other hand, they were initially a little further off than the stable distance they moved into the stable distance when the air was set in vibration. This experiment disproves conclusively the old theory of ridge formation, for by no modification can a repulsion varying inversely as the fourth power of the distance account for the attraction which takes place when the distance between the spheres exceeds a certain value.

If more free spheres were placed in the neighbourhood of the fixed row they added themselves above both the fixed and the free row to make little walls, as shown in fig. 21, Plate 23. If, by tapping the tube, a wall of free spheres was made to build up on the row of fixed spheres, and the tube was then rotated about its axis to the position shown in fig. 22, Plate 23, the free spheres remained in position. Even if the row of fixed spheres was brought to be at the top of the tube the free spheres remained attached as long as the air vibration continued, as shown in fig. 23, Plate 23, although they dropped the moment it was cut off. This is of significance in connection with the antinodal disc (see Section 8). A few free spheres which happened to be in the bottom of the tube arranged themselves in a wall in the same plane. The lateral adhesion of the spheres is further illustrated in fig. 24, Plate 23, which shows an isolated wall of free spheres which built itself up.

5. *Forces between Obstacles which are the Centre of Vortex Motion.*

In connection with the formation of dust figures, experiments were carried out to obtain a general understanding of the forces between obstacles which are the centres of vortex motions of the type discussed in the paper on "Circulations," Section 5. The water-jacketed box and tubes shown in fig. 6 of that paper were used. A hollow cylinder of ivory 3·1 cm. long, 0·15 cm. in diameter and weighing 23 milligrams was suspended as shown in fig. 1 (*a*), being attached to one end A of a rigid arm AB, to the other end B of which a small counterweight was fixed. The suspension was an annealed platinum wire of 8μ diameter, and about 11 cm. in length, which was carried by an attachment fixed to the top of a suitable glass tube. The tube was mounted on a brass frame-work cemented to the roof of the box, constructed so that the whole suspension could be displaced longitudinally as desired; the suspension wire itself passed through a fine longitudinal slit in the roof of the box. The upper attachment of the wire was capable of rotation, so that the suspended cylinder could be arranged to be in equilibrium at any required position. Another cylinder of ivory, of the same external dimensions, was prepared. Smoke was introduced into the box, so that the vortex formation could be observed at the same time as the nature of the forces. In all cases frequency 512 ∼ was used, and the tube was adjusted so that the two cylinders were in the antinodal region.

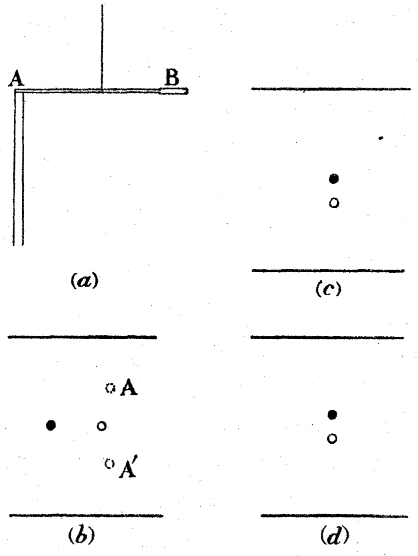

FIG. 1.

(*a*) The suspended cylinder to investigate the forces between cylinders.

(*b*), (*c*), (*d*) Different positions of the suspended cylinder ○ and the fixed cylinder ●.

For the first series of experiments the one cylinder was fixed rigidly in the middle of the box, and the other arranged longitudinally opposite to it, as shown in fig. 1 (*b*), the black disc indicating the fixed cylinder and the continuous outline the suspended cylinder. In these positions attraction or repulsion between the cylinders was indicated by the suspended cylinder tilting towards or away from the fixed cylinder, while transverse forces led to a movement producing pure torsion of the wire. For all distances less than a certain critical distance, when vortex motion formed in the air round the cylinders the longitudinal position was unstable, the moving cylinder being thrown to one side or the other. Equilibrium between the torsional force of the suspension and the mechanical force between the cylinders could be reached in some such position as A or A′, the distance of these points from the central line depending upon the intensity, increasing as this increased. This effect took place whether the cylinders were at such a distance that both vortex systems were distinctly formed, or were closer, so that only one formed well at the particular intensity used. The critical distance to which reference has just been made is one where the vortex systems originated by the two cylinders just touch. When this is the case there appears to be no marked instability, the cylinders remaining in position opposite one another.

A first indication of the longitudinal forces which exist if the cylinders are constrained to remain both in the plane of the vibration vector can be obtained by watching the behaviour of the cylinders when the air is first set in vibration. If the cylinders are very close, so that the distance between their nearest points is about a diameter or less, the suspended cylinder tilts towards the fixed cylinder before it is displaced sideways, showing that there is an attraction at small distances. If the cylinder were completely free it would move towards the fixed cylinder and laterally, taking up a final position close to and at the side of the fixed cylinder. While there is an attraction at such small distances, for greater distances, up to the critical distance, there is a repulsion between the cylinders, as evidenced by a movement apart before the lateral movement takes place. When the distance is greater than the critical distance there is no lateral instability. The method of suspension was not very suitable to show small repulsive or attractive forces, but if the distance slightly exceeded the critical distance there were definite signs of attraction tending to make the vortex systems touch. Clear evidence on this point is afforded by experiments with the cylinders at the side of the box, described a little later.

In another series of experiments the equilibrium position of the suspended cylinder was to one side of the fixed cylinder, as shown in fig. 1 (*c*). In this case the controlling factor appears to be the size of the vortex system initiated by the fixed cylinder, *in the absence of a second cylinder*. If the equilibrium position of the suspended cylinder is inside this vortex system, it is drawn in and held against the fixed cylinder as long as the air is maintained in vibration. If the equilibrium position is just outside the vortex system, the cylinder is repelled to a distance such that the vortex systems formed by the two cylinders just touch. The vortices are slightly deformed by the transverse force exerted by the torsion of the suspending wire.

To see if the size of the box had an undue influence, the transverse experiments were also tried with the fixed cylinder displaced somewhat from the central plane of the box, so as to give the suspended cylinder more room, as shown in fig. 1 (*d*). Exactly the same phenomena were observed. The vortex system formed by the suspended cylinder was well clear of the side of the box.

S. R. Cook,* working with spheres of sealing-wax of about 0·2 mm. diameter, found that while in general he obtained repulsion between spheres whose line of centres was

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* 'Phil. Mag.,' vol. 3, p. 471 (1902).

parallel to the vibration vector, yet, when the distances apart were less than half the diameter, the suspended sphere approached the fixed sphere and moved round until the line of centres was perpendicular to the stream line. This is in accordance with our observations. However, when the line of centres was transverse to the vibration vector he found that, when the distances apart were within about half a diameter the spheres attracted, but that when they were closer than this they repelled one another. This observation is in clear contradiction to what is here found. I suggest that the repulsion must have been due to electrification, a suggestion which is supported by COOK's observation that the effect was more noticeable with glass than with sealing-wax spheres. A pure hydrodynamic effect would clearly not be influenced by the material of the spheres. COOK, of course, did not know of the existence of the vortex motion, and had no satisfactory explanation of the various attractive and repulsive effects. His conclusion was that "a perfect fluid contains forces which are essential, but are not sufficient, to produce laminæ in the form in which they exist." Reference to the experiments of GEORG THOMAS has already been made in the introduction to this paper.

Our experiments show, then, that when two cylinders (or spheres), with their joining line oriented in any direction, are close together one vortex system is formed round the two, and they move so as to come together, with their line of centres transverse to the vibration vector. At larger distances the two cylinders tend to repel one another, so that each can form its own vortex system. The distance at which attraction takes place is small for the longitudinal position, but much greater for the transverse position. This might be expected from the direction of the circulation, illustrated in fig. 7 of the paper on "Circulations."

To investigate the conditions leading to the formation of ridges, we must consider what happens when the cylinders are close to one side of the box, corresponding to lines of dust on the bottom of the tube. In the first experiment a cylinder (a steel rod 0·317 cm. in diameter) was fixed rigidly to a vertical wall of the glass box. When the air was set in vibration a vortex system formed about the cylinder which appeared to be exactly half the system as formed by a cylinder set in the middle of the box. (See, *e.g.*, "Circulations," fig. 21, Plate 9.) The loop to the right of the right-hand cylinder of the two shown in fig. 27, Plate 23, gives a good idea of half the system formed by a cylinder fixed to the wall; the loop to the left of the cylinder in the picture is slightly distorted by the presence of the second cylinder. This vortex system of two loops only might be expected since, there being no flow across a plane through the axis of the cylinder parallel to the vibration vector, this plane could, apart from a small viscous drag, be replaced by a rigid wall.*

Two cylinders of 0·317 cm. diameter were then fixed parallel to one another to a vertical wall, with their axes 1·5 cm. apart. At high frequencies each cylinder formed its own independent vortex system. As the frequency was lowered, with

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* The rigid wall in the experiment is displaced from this position by a distance equal to the radius of the cylinder, but the displacement has apparently no marked effect on the motion of the air.

attendant increase of amplitude of the air motion, the length of the tube being adjusted to resonance in each case, the vortices grew in size until the systems touched. Further lowering of frequency caused one system to grow at the expense of the other, the larger loop pushing the smaller to one side, it being a matter of chance which side grew the larger vortex. A similar effect to that produced by changing the frequency could be produced by changing the amplitude at a fixed frequency.

The state when the vortices touch is shown in fig. 26, Plate 23. A similar picture for smaller cylinders, diameter 0·167 cm., is shown in fig. 27, Plate 23. It was concluded, from the experiments with the two cylinders in the middle of the box, that this state of the vortices corresponded to the stable system with no attractive or repulsive forces between the cylinders. To prove this the suspended ivory cylinder was again used, being now brought close to the wall, with the suspending rod transverse, so that a longitudinal motion of the cylinder was controlled by the torsion. Since a clearance of about half a millimetre could not be avoided, the fixed cylinder was also brought about the same distance from the wall. This leads to two very small vortex loops being formed between the cylinder and the wall, as seen in fig. 28, Plate 23, where, incidentally, the optical reflection of the system in the glass wall also shows well.

The suspended cylinder being in equilibrium it was found that, while high frequency, with its attendant small vortices, led to attraction, and low frequency with its attendant large vortices to repulsion, at a frequency which formed the vortex pair just touching, as illustrated in figs. 28 and 29, there was no motion of the cylinder when the vibration was switched on and off. In these pictures the right-hand cylinder is free to move. This, therefore, represents the state of things in an equilibrium position and corresponds to the formation of stable ridges or to the formation of a single line of free pith balls parallel to a fixed line of balls, as shown in fig. 20, Plate 23.

These experiments show that, with two obstacles in the longitudinal position, there is an attraction when the distance between them just exceeds the critical distance at which the vortex systems touch, a repulsion when the distance is less than this, while, when the obstacles are very near, there is always an attraction.

The mode of formation of the small intermediate ridges of dust shown in figs. 7 and 9, Plate 21, is now clear. Some of the smaller particles of cork are swept round by the vortices and deposited in the position I midway between the two ridges of dust R_1, R_2 which take the place of the cylindrical obstacles (fig. 2). This continues until sufficient cork has been deposited for the light intermediate ridge to form its own

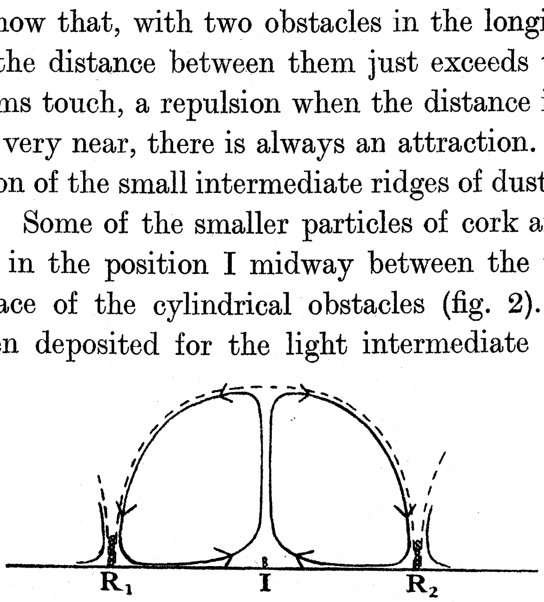

FIG. 2.—To illustrate the formation of the intermediate ridges.

marked vortex system, when the intermediate ridge behaves much as a full ridge, as observed. That the particles which form the intermediate ridge are, in fact, on the whole smaller than the particles of the main ridges was proved by collecting particles from both types of ridges when the appearance was as illustrated in fig. 7, Plate 21. As a result of several measurements, the mean diameter of the particles of the main ridges came out to be 0·071 mm., and of the intermediate ridges 0·054 mm.

6. *The Formation of the Eyes.*

As already described, the formation of eyes takes place at a frequency a little below that which gives the antinodal discs. As in the phenomenon of ridge formation, the energy of the vibration plays a controlling part. If this energy is large enough, then the movement to the nodes which leads to the formation of eyes takes place even at the precise frequency which forms antinodal discs; these remain in position, although in consequence of the small number of particles and the large amplitude, their outline is very blurred. The dust is entirely removed from their surroundings. It is clear, then, that no particular importance can be attached to the precise difference of frequency which characterises the two states, but the following will serve as an example of what often occurred. The numbers give the frequencies at which antinodal discs were observed to be well formed, and at which the clearance leading to the formation of eyes was observed. The absolute frequencies are very rough, the difference of frequencies tolerably accurate.

Antinodal discs	275	420	460	710	845
Eyes	272	413	449	703	840

The method in which the dust is carried to the nodes has been described in Section 3. This can be explained in terms of the general node-to-antinode circulation described in the paper on "Circulations," and the vortex formation round an obstacle. If conditions are such that the general circulation is very strong, the flow near the walls will carry particles along with it, the particles most easily entrained being the extreme particles of each ridge, since at the middle of the ridge, where it is several particles deep, the inter-ridge vortex motion will be more vigorous, as can not only be deduced from the fact that vortex motion is more easily initiated by a big obstacle (see "Circulations"), but can also be directly confirmed by observation of the motion of the minute particles which are always to be found mixed with the larger cork particles.

Again, the velocity of the general circulation is greater close to the wall than it is at some little distance from it,* so that at the extremities of the ridges, where the flow is undisturbed, the motion due to the general circulation is largest. The ridges near the antinode are therefore robbed of their particles from the edge, and gradually destroyed, while fresh ridges are built up near the node, and ultimately carried towards it. A general confirmation of this view is given by the fact that the more dust there is in the tube, the more stable are the ridges, and the less the general motion towards the nodes.

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* See "Circulations," p. 454.

Experiment having shown that the dust is not carried right up to the node, but deposited in a ridge which constitutes one side of the formation which we have called an eye, the question arises as to where the process of carrying towards the node will stop. It appeared likely that the position of the walls forming the eyes, which indicates the termination of this travel, corresponded to the point at which the amplitude of the air motion, at the given frequency, is just sufficient to cause vortex motion round the particles. To test this hypothesis the eyes were formed with different intensities of vibration. As was to be expected, the feebler the intensity, the wider was found to be the spacing of the two sides of the eye. Figs. 31, 32 and 33, Plate 24, show the results which were obtained with three different intensities, in diminishing order. These pictures were obtained with a frequency of 538 ∽.

To obtain a quantitative check the spacing of the eyes was measured with five different intensities, and the maximum double-amplitude of vibration measured in each case at the antinode by observation of the movement of very small cork particles, drawn out into lines. These cork particles were too heavy to partake of the full motion, but an estimate of their size was made by observing their rate of fall when the sound was cut off, the density of the cork being taken as 0·24. By the method described in the paper on "Circulations," the relation of the amplitude of the particle vibration to that of the air was calculated. There may be an error of a few per cent. involved in this process, but for the present purposes high accuracy would have been out of place, in view of the uncertainty in the other factors. The double-amplitudes so found were plotted against the resistance which was put in circuit with the oscillator unit to diminish the intensity, and the best smooth curve put through them, from which the double-amplitude corresponding to a given resistance, taken as the parameter, was read.

Assuming a sine-wave form, which was approximately the case, as detected by the cathode ray oscillograph, the double-amplitude, at the place where the edge of the eye was formed, was calculated in each case from the measured double-amplitude at the antinode. This was done instead of reading the double-amplitude at the edge of the eye directly, as the latter was too small for even moderate accuracy to be possible. The following table embodies the results. The resistance given in the first column is used

TABLE II.—Frequency 840 vibrations/sec., node to node distance 40·2 cm.

Resistance in circuit (in 10,000 ohms).	Average distance of eye wall from node.	Double-amplitude at antinode $= a$.	Sine factor $= s$.	Double-amplitude at eye wall $= sa$.
	cm.	mm.		mm.
0	1·02	0·335	0·155	0·052
1	1·52	0·245	0·232	0·057
2	2·87	0·170	0·427	0·072
3	4·37	0·110	0·621	0·068
4	5·75	0·070	0·775	0·054
			Average ...	0·061

as a label to indicate the particular experiment and is not involved in the calculation. The sine factor *s* gives the ratio of the double-amplitude at the edge of the eye to that at the antinode, on the assumption of sine-wave form.

It will be seen that the results at different intensities give approximately the same amplitude at the ridges constituting the eye, although the width of the eye varies from 2 to 11·5 cm.

Measurements were made to ascertain the amplitude necessary, at the given frequency, to produce a well-marked vortex motion, with an obstacle of the size in question. Cork was collected from the eyes and the average diameter of the particles found to be about 0·082 mm. A line of such particles was assumed to have about the same effect in initiating vortex motion as a cylinder of this diameter. A gold wire of diameter 0·0032 inch = 0·0081 cm., which is a standard size, was fixed across the water-cooled box shown in fig. 6 of the paper on "Circulations," and smoke was introduced. The intensity was then altered by varying the resistance in circuit with the oscillator unit until the minimum was found at which a well-marked four-loop vortex system developed. The double-amplitude corresponding to this resistance was then measured with smoke particles, as described in the paper on "Circulations." Independent readings by two different observers, repeating the whole experiment, gave 0·078 mm. and 0·068 mm., which, considering the arbitrary element involved in the judgment of the point at which the vortex system is well developed, is satisfactory. The mean double-amplitude was therefore 0·073 mm., which agrees as well as can be expected with the 0·061 mm. found from the eyes. There is little doubt, then, that the view of eye formation put forward at the beginning of this section is correct.

A series of experiments on eye formation was also carried out in the square tube described in Section 7. The eyes then become simply two straight ridges, one on each side of the node, as shown in fig. 36, Plate 25. The amplitudes were assumed to be in the same ratio for the different currents through the oscillator unit as they were at the frequency for which data are given above. The results, which serve to show that the position at which the ridges form with a given frequency is determined by the amplitude alone, are as follows:—

TABLE III.

Average distance of eye ridge from node.	Maximum double-amplitude (Arbitrary units).	Sine factor	Double-amplitude at ridge. (Arbitrary units).
cm.			
0·65	1	0·061	0·061
0·93	0·73	0·087	0·064
1·37	0·57	0·129	0·066
		Average ...	0·064

7. *The Spacing of the Ridges.*

Various attempts* have been made to establish quantitative regularities in the spacing of the ridges, with the object of confirming deductions from KOENIG's formula, or adaptations of it. It seemed worth while, with the improved technique at our disposal, to attempt some measurements of greater precision than those previously carried out.

All the previous results were obtained from the patterns deposited after the note had ceased. ROBINSON's results showed so little regularity that he was led to divide the system of ridges between two nodes into three parts, left, middle and right, and take the mean spacing of each part. The left and right readings should, of course, be the same. In the results quoted in his second paper, where he put forward a revised theory, the actual means are 1·01, 1·25, 1·07 mm. He takes the mean of the left and right, and then the ratio of this mean to the middle value, which is, for example, 0·83 in the case just given. The values which he obtains with different powders and different frequencies are 0·94, 0·83, 0·80, 0·70, 0·78, 0·89, 0·66, 0·70 (which, on his theory, should all be the same), and of these he takes the mean, *i.e.*, 0·78, for comparison with his adaptation of KOENIG's formula. According to different assumptions as to the extent of the ridge system, he gets theoretically 0·60, 0·65, 0·76 for the ratio, and says that, " considering the ripples to extend only from 0° to about 70°, which is frequently the case,† there is very good agreement between the experimental results and the law $\cos^{2/3}\theta$ deduced from KOENIG's theory "—a statement which, considering the irregularity of the experimental results, shows a certain generosity. IRONS says that, partly owing to the presence of minor ripples, his experimental results are irregular and that the only conclusion justified is that, in general, the distance between striæ increases on approaching an antinode, and adds, " it would perhaps be better to regard the work as failing to support ROBINSON's theory rather than as disproving it."

There are two features in which the origin of irregularities may be sought: one that the tube is circular in cross-section with the result that the ridges grow shorter and shorter towards the nodes (see *e.g.*, figs. 7, 8 and 9, Plate 21), and the other that the particles are of irregular shapes and sizes. It should also be pointed out that unless the ridge system extends over a large fraction of the distance between the nodes the variation of amplitude from point to point is not considerable. Measurements carried out by the writer on ridges produced with cork particles in tubes of circular cross-section showed no great regularity, although increase of spacing towards the antinodes was clearly evidenced. There were also indications of an increase of the spacing at

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* J. ROBINSON, 'Phil. Mag.,' (6), vol. 18, p. 180 (1909); (6), vol. 19, p. 476 (1910); E. J. IRONS, 'Phil. Mag.,' (7), vol. 7, p. 523 (1929).

† This leads to the value 0·76.

the extreme ends of the ridge system, so that a minimum spacing appeared to exist. This is, no doubt, an effect of the diminution in the size of the ridges and can be clearly seen in, for example, fig. 39, Plate 25, obtained with uniform spheres.

In an endeavour to produce greater regularity of ridges, experiments were carried out in a tube of square cross-section. This was made from drawn brass tube, of internal cross-section 3·5 × 3·5 cm. and 154 cm. long. Opposite faces of the brass were removed at the central portion over a length of 46 cm. and plane glass windows were cemented on, enabling a clear view to be obtained of the behaviour of the dust on the plane floor of the tube.

Cork particles placed in this tube form beautifully sharp and straight ridges, extending right across the tube, as seen in figs. 34 and 35, Plate 25, the former showing three groups of ridges, separated by two nodes, and the latter the central part of one group. The fact that the ridges appear broader to one side than at the middle of the tube is, of course, merely a perspective effect, the walls of finite height being seen vertically from above at the middle, but obliquely at the sides. The phenomena of clearance and of the connected formation of eyes, which in this case consist simply of two straight heavy ridges equidistant from, and to either side of, a node, and of the antinodal disc, occur just as in the round tube. Fig. 36, Plate 25, shows a nodal eye; fig. 37 an anti-nodal disc (or "plate" in this case) in a square tube.

In attempting to obtain perfectly spaced ridges, either in a round or square tube, two difficulties are encountered. Firstly, that it is almost impossible to introduce just such an amount of dust that all the ridges of the system are well-formed, without patches of superfluous dust which disturb the formation of the ridges in their neighbourhood. Secondly, the formation of the intermediate ridge between some ridges, but not between others, since, with powders such as cork, there are always some smaller particles present. It was found possible temporarily to suppress the intermediate ridges by tapping the tube sharply near the point where they are formed, so that the fine powder constituting them is thrown up into the air, caught in the vortices and carried to the main ridges. This is the way in which fig. 8, Plate 21, was obtained.

Measurements made with the ridges shown in fig. 34, Plate 25, show clearly the very regular spacing at the antinodes and that the spacing near the nodes is smaller. They appear, as far as comparison can be made, to show considerably greater regularity than previous measurements, but they were not held to be fully satisfactory. It was decided to attempt the preparation of truly spherical particles of uniform size.

Such particles were made of carnauba wax, which is very hard and has a specific gravity of about 1·0. The wax was sprayed by a special apparatus, consisting of a hollow cone, with holes at the upper edge, which could be rapidly rotated about the axis of symmetry. The lower end of the cone being dipped into molten wax, the liquid rises up the internal sides of the cone in consequence of the centrifugal acceleration, and is thrown out through the holes in the form of fine drops which solidify in the air.

The spherical particles so formed are collected on sheets of paper spread round the sprayer. Smaller particles fall near the sprayer, larger ones farther off. The faster the rotation the finer the particles. Particles of approximately uniform size were collected at a given distance, and further graded by sieves. When greater uniformity was required the final grading was done by a long adjustable slit in the bottom of a cylindrical vessel. This being set at a given width, a large quantity of sieved particles was placed in the vessel, which was then agitated. Particles passing through the slit were collected, and passed through the vessel again, the slit having been slightly narrowed. Particles remaining in the vessel after prolonged agitation were then nearly uniform in diameter. Fig. 38, Plate 25, shows a magnified view of sample of particles so obtained, which were about 0·29 mm. in diameter.

A small quantity of wax spheres placed in a circular tube gave a beautiful series of ridges, as shown in fig. 39, Plate 25. The shadows to be seen to the right of each ridge show how clearly separated the particles in the ridge are. The particles in this case are ones graded by sieving only, and were about 0·6 mm. in diameter. It is to be noted that there is no tendency to form intermediate ridges; this is to be expected, since this phenomenon has been shown to depend upon the presence of particles of widely different sizes. Electrification of the particles being suspected, the tube was silvered and polished inside, and the silver coating earthed. It is doubtful whether real electrification effects were present in the unsilvered tube, the irregularities being afterwards traced to other causes.

Since the spacing in the cylindrical tube is clearly influenced by the fact that there are very few particles in the extreme ridges, experiments were then carried out with wax spheres in a square tube. Very uniform particles graded with the slit, of diameter about 0·29 mm., were used in these experiments. Ridges are formed across the tube, but with the wax spheres they are not quite so straight as they are with the cork particles. The wax spheres move with great ease on the smooth surface of the glass, so that the slightest disturbance will impair the straightness of a ridge. The great straightness of the cork ridges may be due to the friction, which undoubtedly exists in the case of cork particles, having a stabilising influence, or it may be that the irregularly shaped particles in the ridge become partly entangled, and so form a flexible strip which tends to behave as a whole. Be that as it may, the extreme mobility of the wax spheres has as a result that the ridges react to the slightest irregularity in the motion, and present the appearance shown in fig. 40, Plate 25. (It is to be regretted that two better photographs of ridges of wax spheres, which showed greater regularity and were used for the measurements of fig. 3, were destroyed in an accident before prints were made.) In spite of the fact that the ridges are not quite straight, consistent measurements of the spacing can be made by measuring at places where the ridges run for some little distance parallel. The results so obtained show a much greater regularity than any of those obtained with cork, either by the present writer or by previous workers.

Fig. 3 shows the spacing of the ridges plotted against the distance measured along the tube. Two sets of measurements obtained by two independent observers are shown: in the one case, shown by the crosses, the spaces were divided into consecutive fives and each five averaged; in the other case, indicated by circles, they were divided into fours and the fours averaged. The frequency was about 720 ∼. It will be seen that the experimental points lie pretty well on the smooth curve, which represents the empirical formula

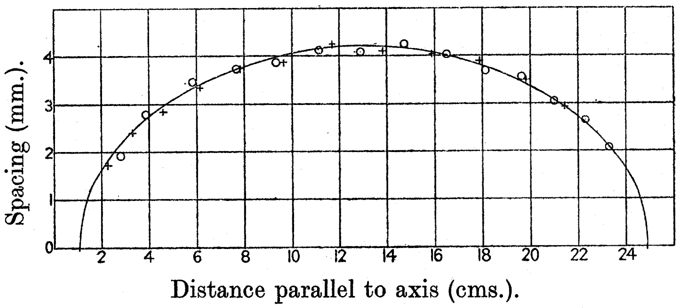

Fig. 3. The spacing of the ridges.

$$s = 0{\cdot}422 \cos^{0{\cdot}44} \frac{y}{y_0}\pi$$

where s is the spacing between the ridges in cm., y is the distance of the ridge concerned from the antinode, and y_0 is the distance between two nodes. Robinson obtained, from Koenig's theory, formulæ in which the spacing was proportional to $\cos^{4/3} \frac{y}{y_0}\pi$ (original theory) and to $\cos^{2/3} \frac{y}{y_0}\pi$ (revised theory). Our measurements allow us to fix the index to within about 2 per cent., which shows once more that Robinson's theory does not represent, even approximately, the experimental facts.

The spacing actually depends on the size of the vortices. The dependence of the size of the vortex on the amplitude of the air motion, other things being equal, has not been measured.

8. *Explanation of the Dust Phenomena.*

The phenomena to be explained are:

1. The characteristic differences between the behaviour of heavy particles and light particles, originally noted by Kundt.
2. The characteristic differences between the dust figures formed when the vibrating diaphragm is at a node or at an antinode, as noted by Kundt.
3. The appearance of sharp ridges.
4. The appearance of intermediate ridges.
5. The waviness of the ridges when much dust is present.
6. The appearance of the antinodal disc.
7. The formation of the eyes, and of the single wide ridge at the node, which is merely a particular case of the eyes.

The variable effects which influence the phenomena are, at fixed tube length, the frequency and the intensity. The wave form does not appear to have much influence, but in some cases the experiments were, for definiteness, carried out with a pure sine-wave form. It is to be particularly noted that with a diaphragm driven by an oscillating set and attached to a tube of given length, changes of frequency always lead to marked changes of intensity, especially in the neighbourhood of a resonant frequency, and, that, in addition, the intensity is different at different resonant frequencies. Care is therefore needed to distinguish those changes which are due to changes of frequency, *qua* frequency, from those which are due to the associated changes of intensity. In particular, the changes which KUNDT noticed according as the driving disc was at a node or at an antinode, are due solely to differences of intensity.

Two phenomena of prime importance for the explanation offered are the general node-to-antinode circulation and the vortex motion round the individual particles, both experimentally demonstrated for the first time in the paper on "Circulations."* It is important to note that the vigour of the vortex motion depends upon the motion of the surrounding air relative to the particle, so that with a light particle of given size, which partakes of the motion, a much feebler vortex is formed than with a heavy particle, which remains at rest.

The variation in the relative vigour of the two types of air motion, general circulation and vortex motion, is also important for the discussion. It has been established, as described in Section 5 of the paper on "Circulations," that when the intensity of vibration of the air is high the general circulation so far prevails that no definite vortex is formed round an obstacle. As the intensity is lowered a point is reached at which a large well-defined vortex is formed in spite of the (much enfeebled) general circulation. If the intensity is further decreased the vortex shrinks and its internal circulation

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* These two phenomena were briefly described, with pictures, in a communication to 'Nature,' March 21, 1931. Some weeks after this I received a communication from Mr. ROLLA V. COOK, of Bethany College, U.S.A., drawing my attention to a brief note of his which emphasises the advantage of using sunflower pith in KUNDT'S tube, published in three places, viz., 'Nature,' vol. 118, p. 157 (1926); 'Science,' vol. 64, p. 404 (1926); and 'Sch. Sc. and Math.,' vol. 26, p. 722 (1926). This note contains the sentence, "By the use of this powder I was able to obtain discs that extended completely across the tube," which from the context can only mean that he made the ordinary ridges extend right across the tube, which is easily done with tubes somewhat narrower than those used by me. There is no mention of an antinodal position—in fact, the above sentence is the sole mention of the discs—and it is doubtful if true antinodal discs can be obtained with the unsteady conditions elicited with a stroked rod, which Mr. COOK used to excite the tube. In a letter dated April 15, 1931 (*i.e.*, over three weeks after my letter was published in 'Nature'), published in the 'Physical Review,' vol. 37, p. 1189 (1931), Mr. COOK gives a diagram closely resembling a photograph published in my letter, and suggests that the ridges are due, as stated in my letter, to air vortices, without making any reference to my publication. The direction of circulation was not given in my letter, and Mr. COOK shows it the wrong way round in his diagram. Mr. COOK'S suggestion, that the striæ are formed in the same way as ripple marks in sand, is sufficiently disproved by, *e.g.*, the experiment with pith balls here described.

becomes less vigorous. Finally, the vortex becomes less definite in outline until, in the end, no vortex is formed. Already at the state shown in fig. 23, Plate 9, of the paper on "Circulations," the general circulation is weak, although no rule can be laid down as to which, general circulation or vortex, disappears first, since the minimum intensity necessary for vortex motion depends on the size of the obstacle as well as on the frequency and amplitude. We may summarise the points just discussed by saying that, as the intensity of the sound increases, the general circulation gains in strength relative to the vortex motion round an obstacle.

The intensity has maxima at the resonant frequencies, *i.e.*, when a node is formed at (strictly speaking a short distance behind) the diaphragm.* When the intensity is already large in itself a very vigorous general circulation (from the antinode to the node along the walls, returning up the centre) takes place at resonance, and carries with it all the dust which lies on the bottom of the tube to the nodes, where it eventually deposits as a narrow heap. Some small portion of the dust may be temporarily carried up the centre by the return flow, but as any particle which once gets to the neighbourhood of a node remains there, the whole of the dust is eventually deposited there. If the powder is light the particle oscillates with a considerable fraction of the full amplitude of the air oscillation, and there is only a slight tendency to form the vortices round particles to which steady ridge formation is due. Light particles are also more easily carried along than heavy ones. For both these reasons, as also because at large intensities the general circulation is the predominant effect, the powder is carried as described. KUNDT's observation that light powders form heaps at the nodes when the driving disc or diaphragm is at a node is thus explained as an intensity effect.

If the frequency and tube length are such that an antinode is formed near the diaphragm the intensity of the air vibration is small.* The vortex motion, though diminished, is less affected by diminishing intensity than the general circulation, relative to which it becomes stronger and more stable. We then have conditions described in Section 3, and, in accordance with the discussion in Section 6, eyes are formed, the width of the eye depending upon the strength of the vibration. This is the explanation of KUNDT's second observation with light powders, viz., the formation of "holes" at the nodes. When the tube is excited by KUNDT's method no ridges are seen after the vibration has ceased, because the vortex motion is still weak, and, owing to the small inertia of the particle, the dust is carried to the node during the stroking of the rod.

When denser particles are used, *i.e.*, particles of high inertia, the vortex motion round the particles is very vigorous compared to that initiated by a particle of small inertia under similar conditions. With a vigorous oscillation of the air, that is, with a node at the driving disc, ridges are formed by the vortex motion, as discussed in Section 5. If the air oscillation is feeble, *i.e.*, with an antinode at the driving disc, there will be feeble or

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* This point will be discussed in a forthcoming publication.

no vortices and feeble or no general circulation, and so the particles remain at rest. Thus KUNDT's observations with sand and iron filings are explained. All KUNDT's observations can be repeated with one kind of powder, namely, cork particles, the control put at our disposal by the technique described.

When experiments are carried out in the range of medium frequencies, by varying the frequency, as described in Section 3, sharp ridges appear over particular small ranges of frequency on either side of the resonant frequency. This is an intensity effect, these particular frequencies corresponding to the intensities that produce a well-marked vortex motion which maintains itself in spite of the general circulation, as illustrated in the case of cylinders by figs. 26 to 29, Plate 23. At resonant frequency the intensity is increased and an antinodal disc is formed; the ridges maintain themselves but are not so sharp, owing partly to the increased intensity of vibration and partly to the increased general circulation. The way in which sharp ridges can be produced at resonant frequency, if they are blurred when the full intensity is applied, by cutting down the intensity, is another indication that sharp ridges can be obtained at any frequency if the intensity is adjusted to a certain optimum. When the intensity is too high the antinode-node circulation is relatively strong and we have clearance to the nodes; when the intensity is too low the vortices are not well formed, and we have feeble or no ridges.

In the range of low frequencies sharp ridges are never formed. The lower the frequency the more the particles are carried backwards and forwards by the excursion of the air, with an amplitude which tends towards the full amplitude of the air. The consequences are that, while there is sufficient relative motion to engender the vortices which cause the spacing, the particles which constitute the ridges are always in axial vibration, which leads to the type of appearance shown in fig. 10, Plate 21. At these low frequencies the general circulation is relatively feeble and clearance cannot be produced.

As regards the formation of the individual ridges, the attraction which takes place between obstacles which are placed with the line joining them normal to the vibration vector, and not too far apart (see Section 5), holds the individual particles together into a wall. Gravity keeps the top particles pressing on those beneath, and so stabilises the system. If two particles are floating in the air each one will be the centre of a vortex system similar to that which was demonstrated, in the paper on "Circulations," to hold round the sphere. The experiments already described on the forces between obstacles have shown that two particles lying with their joining line parallel to the axis of the tube arrange themselves at a fixed stable distance if they are constrained to maintain their axial position, but that without such constraint this position is unstable, the two particles tending to move round one another and to approach, so as to lie side by side across the tube. When the particles are, by this mechanism, formed into a complete ridge on the bottom of the tube there is a cylindrical vortex to either side, as shown in the smoke pictures of Plate 23. A second ridge being formed, the two will space themselves so that the vortices just touch as described in Section 6.

The formation of the subsidiary ridges between the main ridges is due to lighter particles being swept by the vortices to the places where they abut, as explained in Section 5.

In the body of a single ridge there is no tendency for an individual particle of the ridge to move transversely, since there are, in general, particles to either side of it, the extreme particles being constrained by the sides of the tube. There is, however, a possibility of the uppermost particles being detached by the circulation, and it is to be noted that once the particle is well separated from the others it is repelled transversely. Should the vibration be vigorous enough the particles can be seen passing from the top of one ridge to another. Gravity, however, exercises a stabilising influence by aiding the force of lateral attraction and tending to keep the uppermost particles in contact with the others.

Imagine now a particle suspended in the air near the upper part of the walls of the tube. Any second particle in its neighbourhood will place itself side by side transversely across the tube, and a transverse wall of particles will tend to form at the wall as illustrated by the case of the pith spheres depicted in fig. 23, Plate 23. Walls of cork particles formed on this principle can be seen in fig. 15, Plate 22. Such a ring can form at any place where the air vibrations are marked, anywhere, that is, not too remote from the antinode. Particles at the tube wall form natural starting places for the ring, for they are restrained by the wall from moving transversely in one direction.

The ring at the antinode will, on account of the maximum intensity of vibration there, be the most stable. Other rings which form at the top of the tube will be spaced from it by the same vortex mechanism as maintains the ridges at the bottom of the tube ; such spaced rings I call upper ridges. There is, however, an essential difference in that gravity is now tending to pull away the particles of the ridge which are farthest from the wall, instead of to hold them down, as in the case of the ridges at the bottom of the tube. The result is that an element of stability is missing. Particles are therefore robbed from the lower parts of the upper ridges farthest from the antinode and built on the more vigorous upper ridges nearer the antinode, this process going on until all the particles are in the antinodal disc.

Successive stages of the building up of an antinodal disc can be seen in figs. 15, 16, 17 and 18, Plate 22, the completed disc being shown in fig. 11. The disc is, for the reason given above, stronger at the wall, and so is of the nature of a well-defined ring filled in with a fainter disc. The whole mechanism of formation of ridges at the top of the tube is the same as that of those at the bottom except that, as explained, they have only a temporary existence, owing to the effect of gravity. Occasionally (probably owing to local irregularities in intensity) two discs will form near an antinode each with accompanying subsidiary discs, as shown in fig. 19, Plate 22, but they always move together to form a single disc at the antinode.

While these are the main forces building up an antinodal disc, the general circulation, two loops of which abut at an antinode, also has a minor influence, tending to remove particles from the disc. As a result of this and gravity there is always a confused movement about the lower part of the disc as can be seen in fig. 12, Plate 22, the circulation at the top of the neighbouring ridges and the forces between the individual particles all playing their part. The top of the discs is, however, perfectly clear cut, as can be seen in the same figure.

The waviness of the ridges when there is much dust in the tube is a consequence of the general circulation. When there is plenty of dust, the ridges towards the antinode are large, being, say, in the case of a tube 3·5 cm. in diameter, nearly a centimetre high at the middle. Now the surface of the reversal of velocity is, as shown in Section 4 of the paper on "Circulations," about 0·58 cm. from the wall of a tube of this diameter, so that the ridges extend upwards, at their mid-point, into the region of reversal of velocity of the general circulation. The wavy and unstable appearance is due to the fact that the middle part of the top of the ridge is tending to be carried by the general circulation in a direction opposite to that in which the general circulation urges the shallow extremities of the ridges. While the locking action of the vortex motion prevents actual transport of the ridges, there is a continual wavy motion of the upper part of the ridge, due to the conflict between the spacing influence and the two opposite directions of the general circulation. The same reversal of the velocity of the general circulation at a certain distance from the walls of the tube leads to the curvature of the ridges seen, *e.g.*, in fig. 30, Plate 24, and to minor phenomena of this nature which need not be further described.

In connection with the initial stages of the motion of a particle from rest on the floor of the tube a paper by Harold Jeffreys* is of interest. He discusses the action of a stream in raising a solid particle lying on the bottom, and points out that, since the velocity of the fluid just above the obstacle is somewhat greater than the general velocity, while at the point of contact it is zero, there must be a region of low pressure above, and of high pressure below, which will tend to lift the particle. The case of a cylinder lying athwart a steady stream is worked out. Similar forces no doubt contribute to the first lifting of the particle in the case of a vibrating fluid, after which it is carried by the circulations in the manner described.

9. *The Behaviour of the Individual Suspended Particles.*

The individual cork particles in the air column show an interesting diversity of behaviour when the diaphragm is set in vibration. The appearance of short lines parallel to the axis of the tube, similar to those obtained with smoke particles and used for the measurement of amplitude,† has already received reference. This is the only type of figure exhibited by the vibrating smoke particles, and hence it is clear that other types of motion shown by cork particles have their origin in the inertia,

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* 'Proc. Camb. Phil. Soc.,' vol. 25, p. 272 (1929).

† 'Proc. Roy. Soc.,' A, vol. 134, p. 445 (1931).

size or asymmetry of the particle, and not in the movement of the air itself. Once more we have an example of the fact that smoke particles are tracing points, while fine cork particles behave in a very complicated manner, largely conditioned by the relative motion of the fluid.

Certain leading features of the behaviour of the cork particles will be described here. Many peculiarities which will not be cited have been explained, but add little to our knowledge of the motion. Between node and antinode particles can frequently be seen which move rapidly in a transverse direction, the path being sinuous, and of limited length. Photographs of such particles show that they are not of sine wave form, as would be expected if they were due to a combination of a rapid rectilinear motion across the tube and a periodic motion along the axis of the tube,* but show in general a crowding up of the crests at either end of the trace. Such traces are shown in figs. 41, 42, 43 and 44, Plate 26, 41 and 42 being obtained with a frequency of 75 and 43 and 44 with a frequency 450, the magnification being about 5·6 and 5 respectively and the exposure 1/8th and 1/50th sec. The axis of the tube is across the page in these four photographs. The form of these paths is easily explained by the consideration that to an observer looking axially along the tube the paths of many particles appear as circles. The actual path of a particle, seen obliquely, is as sketched in fig. 4 (*a*), the terminations of the path being conditioned by the length of exposure. If the motion is viewed normal to the axis of the tube a trace such as fig. 4 (*b*) will be seen if the exposure is such that only half the circuit is completed; a trace such as fig. 4 (*c*), in the case of a circuit nearly completed. Paths of type 4 (*c*) can be seen in fig. 44, Plate 26.

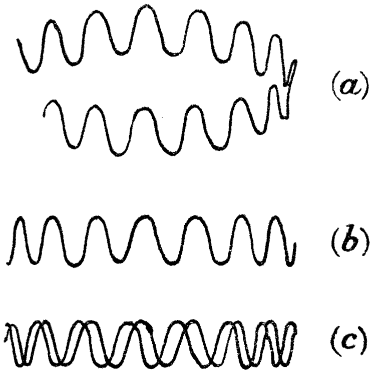

FIG. 4.—To illustrate the motion of the individual particles.

An explanation of the circular paths and other general features of the motion of individual dust particles can be given by making the assumption that the particles are spinning. The particles are not spherical, but possess in general a more or less elongated shape, and will in the first instance set themselves with the greatest length normal to the velocity of the general circulation, as a consequence of forces similar to those which cause a ship or plank to set athwart a stream on which it is allowed to drift.† Since in the general circulation there is a marked velocity gradient, it is natural to suppose that the particles acquire a spin about a longitudinal axis. Such a particle will move under the joint influence of its spin and the (small) velocity of the general circulation relative to the particle, the spin being about an axis lying in a transverse plane, and the velocity being in a direction which varies, but which for the greater

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* As assumed by P. S. H. HENRY, 'Proc. Phys. Soc.,' vol. 43, p. 340 (1931), in his comments on the photographs published by the writer in 'Nature,' vol. 124, p. 724 (1929).

† See *e.g.*, ANDREW GRAY, "Gyrostatics and Rotational Motion," p. 151, etc.

part of the space between node and antinode is approximately parallel to the axis of the tube. (See fig. 5, " Circulations.")

If a cylinder be surrounded by an irrotational circulation, and a pure translational motion of the fluid be superimposed on the circulation, then there will be a fluid pressure on the cylinder which is normal to both the axis and the direction of streaming at infinity. If the cylinder is moving through a stationary fluid the force will be always normal to the path, which therefore, if no other forces are acting, will be a circle,* provided the initial flow is normal to the axis of the cylinder. If it is inclined to the axis the path will, of course, be a cylindrical spiral. KUTTA and JOUKOWSKI† have shown that the same thing holds for any form of contour surrounded by a circulation of the type indicated. Hence, if the particle is spinning with axis across the tube and there is a small relative motion along the tube of the particle and air, the particle will describe slowly a circle of large radius in a plane parallel to the axis of the tube. As the motion is slow, only a very small part of the circle will be shown in a short exposure photograph, and the particle is likely to strike another before much of the circle is completed. Short lengths of path markedly inclined to the axis of the tube can often be observed in the photographs. The closeness of the oscillations, compared to those in the long transverse traces, indicate that the particle is travelling very slowly, as does also the shortness of the trace described during the exposure. Traces showing marked departure from normality to the axis are generally short.

If a spinning particle receives a blow tending to send it across the tube, which must frequently happen on its striking the wall, or may take place wherever the particles are densely distributed, the axis of spin will tend to set itself parallel to the axis of the tube, while at the same time the particle acquires a velocity across the tube. The result to be expected is that the particle should describe a circle in a transverse plane, on which is imposed, as always, a longitudinal oscillation due to the sound vibration. If the blow is violent the circle should be small and described quickly; large circles should be slowly described. If, in addition, there is a general drift of air across the tube at the spot in question the circular motion will be drawn out into a trochoidal motion.

All these figures can be observed in the actual photographs of the dust motion. Figs. 45, 46, 47 and 48 (Plate 26) are pictures of cross-sections of the tube, figs. 45 and 46 at 5 cm. from a node, figs. 47 and 48 at 10 cm. and 19 cm. respectively from a node, the distance from node to node being 38 cm. Circular paths can be distinguished in large numbers in all the figures, especially near the wall. The largest circles are incomplete, the total length of path being in general smaller than that of smaller complete circles or trochoids, which points to a smaller velocity in a large circle. The few irregular paths which can be distinguished in figs. 47 and 48 are well removed

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[*] See *e.g.*, H. LAMB, " Hydrodynamics," 4th edition, pp. 77, 666 (1916).

[†] See W. M. KUTTA, 'Sitz. d. k. Bayr. Akad. d. Wiss.,' vol. 40, " Über eine mit den Grundlagen des Flugproblems in Beziehung stehende zweidimensionale Strömung, 1910," Also *ibid.*, vol. 41, p. 65 (1911).

from the mass of particles and from the wall, in places where the flow of the somewhat distorted general circulation is uniform, so that there is no velocity gradient to spin the particles, and no impact. In figs. 45 and 46, which represent the state near a node, many trochoidal paths can be distinguished, some of which are in focus throughout, while others are in focus at one end only. All these pictures were taken with the tube vertical.

Near the node the general circulation is upward in the region near the wall of the tube, then transverse and radial towards the centre, and downwards in the neighbourhood of the centre. This is in the case where all convection is avoided; in the present instance the circulation is somewhat distorted by convection, the downward current being displaced from the centre of the tube. Where the velocity is transverse the trochoids are in focus all the way. Where it is upwards, but curving in with a radial component, the trochoids are in focus at one end only. Trochoids whose general direction is that of the curve to be expected can be particularly well seen a little to the right of, and above, the centre of fig. 45. In fig. 48, which represents the state midway between a node and an antinode, the successive loops of the few trochoids that are present are very close, as is to be expected in places where the lines of flow of the general circulation are nearly parallel to the axis. The successive loops of all trochoids are surprisingly equal, showing that the velocity is very uniform. This indicates that the head-resistance to the motion of the very small spinning particles is less than might have been expected.

The explanation here offered supposes that the rotating particle is surrounded by an irrotational circulation, and it may be objected that each particle is actually surrounded by a vortex system of the type demonstrated in the paper on "Circulations," and frequently cited in this present account. It seemed, however, possible that the rotation and its attendant circulation would interfere with the formation of the vortex, much as the general circulation does when it is strong. To investigate this point experiments were carried out with cylinders. A mounting was made by means of which a cylinder, protruding through a close-fitting hole in the roof of the glass box of fig. 6 of the paper on "Circulations," could be rotated at any desired speed by a motor, to whose shaft it was axially coupled. A fly-wheel helped to maintain the speed uniform. The rate of rotation was measured by a tachometer of special design, which registered automatically the number of rotations per minute, taken over 15 seconds

The experiments were carried out with three different rods, diameters 0·318, 0·160, 0·087 cm. respectively. The rod was mounted so as to be at an antinode, which is the condition for a vortex symmetrical about a transverse plane through the rod, such as is shown in fig. 22, Plate 9, of the paper on "Circulations." The intensity was maintained constant throughout the experiments with all the rods, and the frequency was 512 ∼ throughout. As the rate of rotation is increased the longitudinal axis of the vortex system tends to turn and set itself obliquely across the tube. At the same time one half of each part of the system originally lying to left and to right of the cylinder, in *e.g.*, fig. 22, Plate 9, of "Circulations," tends to become suppressed, namely, the lower right-hand loop and the upper left-hand loop, when the rotation is anti-clockwise, as seen from above. Increased speed of rotation caused the complete suppression of the loops already diminished, so that in this case there is an oblique S-shaped circulation about the rod. With further increase of speed the simple type of symmetrical circulation associated with a line vortex sets in about the rod.

The speed at which the uniform circulation sets in is fairly definite, and measurements were made of the critical speed with the three different cylinders. Two independent observers agreed to within about 1 per cent. in each case, and the average results were as follows :—

Diameter of rod.	Critical angular speed.
0·318 cm.	333 r.p.m.
0·160	334
0·087	332

These results show that, over a 4 to 1 range of diameter, the angular speed at which the pure circulation sets in is the same, namely about 5·55 revs. per second. If this result can be extrapolated to the kind of diameter which is in question with the cork particles, namely, 0·003 cm., it indicates that when the particles are spinning more than 5 or 6 times a second our assumption of a cylindrically symmetrical circulation is fully justified.

10. *Summary of Results and Conclusions.*

1. The movements of dust particles in the air in a sounding tube can be explained in terms of a general circulation of the air between node and antinode, and a vortex motion round every particle.

2. The vigour of the general circulation gains on that of the vortex motion as the sound vibrations increase in intensity, as a consequence of which the nature of the dust figures obtained is controlled largely by the intensity of the sound.

3. Koenig's theory of ridge formation, which refers the phenomenon of ridge formation to repulsions consequent on a vortex-free motion of the air, is fundamentally unsound.

4. The different types of dust figures obtained with different powders by previous workers, in particular Kundt, can all be obtained with one kind of dust by suitable adjustment of frequency and intensity.

5. The forces between two particles in air, vibrating with an intensity such that they are surrounded by vortex systems of the type described, may be either attractive or repulsive, according to the distance of separation. For particles in the transverse position the force is one of attraction when the particles are within a distance of about a particle diameter, of repulsion at slightly greater distances. The longitudinal position is, at small distances, unstable, the particles tending both to set themselves transversely, and to come together. If the particles are constrained to be in a longitudinal

position there is an equilibrium position when the vortex systems just touch, the forces being repulsive for less and attractive for slightly greater distances.

6. Owing to the lateral attraction and the longitudinal equilibrium position, ridges are formed, which are maintained by the vortex systems at fixed distances apart.

7. When the particles are not of uniform size, intermediate ridges tend to be formed of the lighter particles, which are swept by the air vortices to a position between the main ridges.

8. With uniform spherical particles, made of wax, no intermediate ridges can be formed and the spacing is regular. An empirical formula is given, connecting the spacing of the ridges with the distance from the antinode.

9. When the intensity is large, the general circulation carries the ridges towards the node in a manner described at length in the paper, the dust being deposited in a ridge to either side of the node. At very large intensity these ridges may coalesce to form one broad ridge at the node.

10. At certain intensities a sharp disc extending right across the tube and bounded by a marked ring, is formed at the antinode, the forces which maintain it being similar to those which maintain a ridge at the bottom of the tube, the reversed effect of gravity in the two cases explaining the instability which leads to the formation of one antinodal disc as contrasted with many ridges.

11. The complicated motions of individual particles can be ordered and explained by considering the effect of the spin of the particle in conjunction with the vibration and other movements of the air.

I have much pleasure in acknowledging a grant from the Imperial Chemical Industries Company, expenditure from which was made in connection with this research; the special services of my assistant, Mr. L. Walden, to whose skill the clearness of the photographs is, in particular, due; and the assistance of one of my students, Mr. E. B. Pearson, in connection with the measurements.

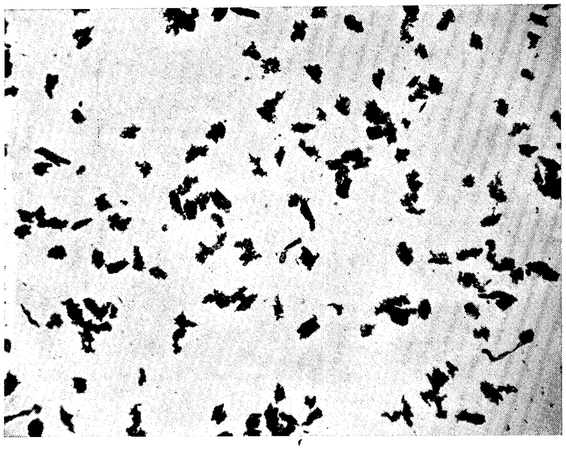


FIG. 5. × 50.

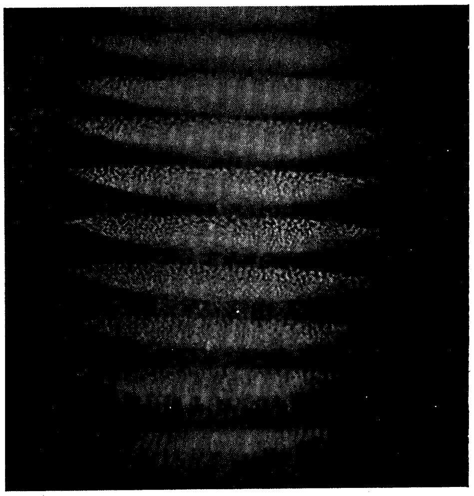


FIG. 6.

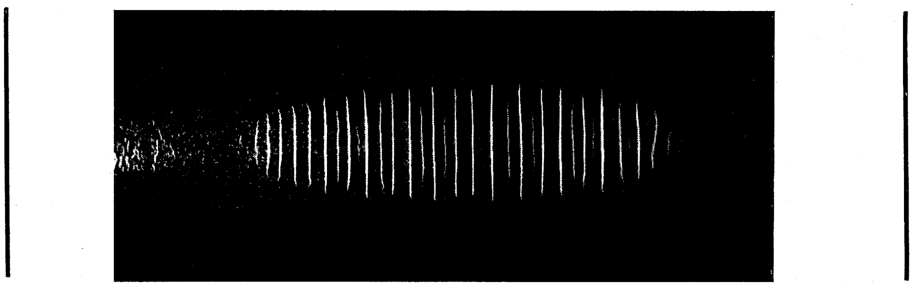


FIG. 7.

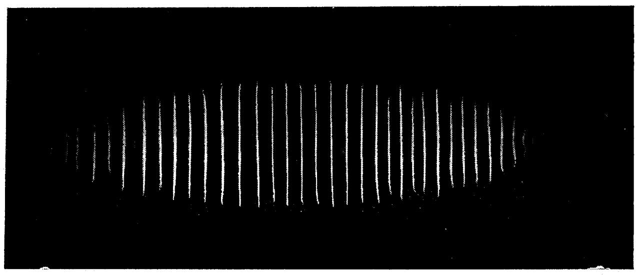


FIG. 8.

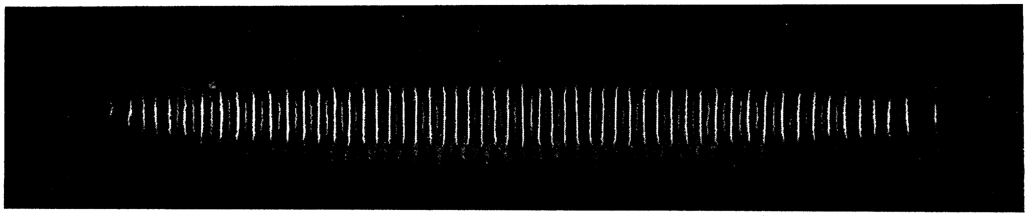


FIG. 9.

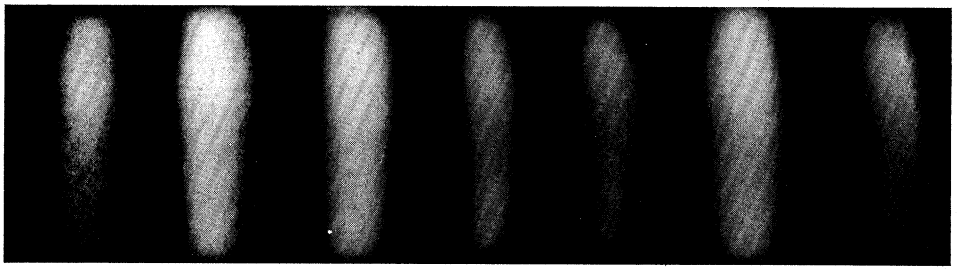


FIG. 10.

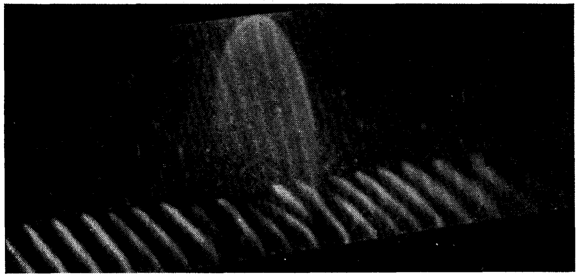


Fig. 11.

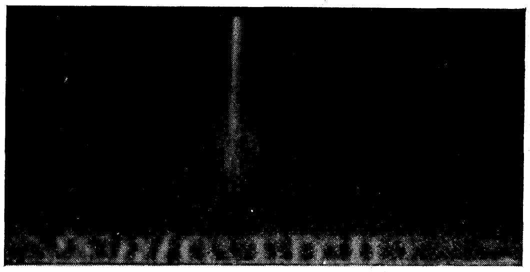


Fig. 12.

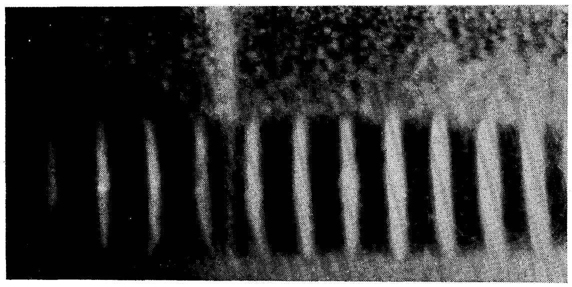


Fig. 13.

Fig. 15.

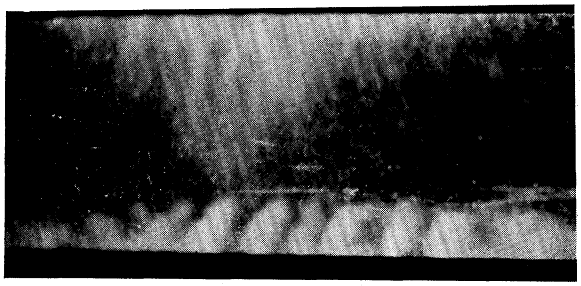


Fig. 16.

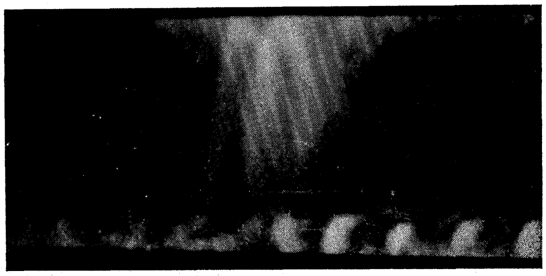


Fig. 17.

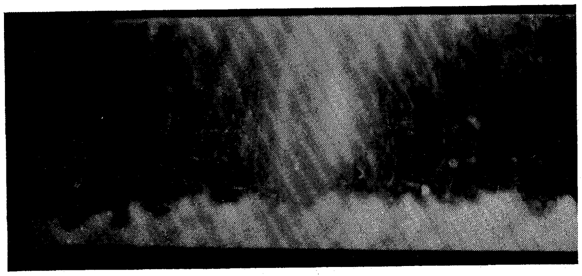


Fig. 18.

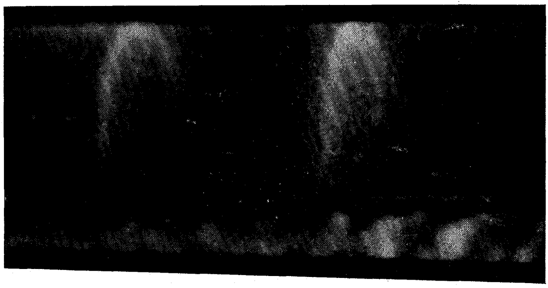


Fig. 19.

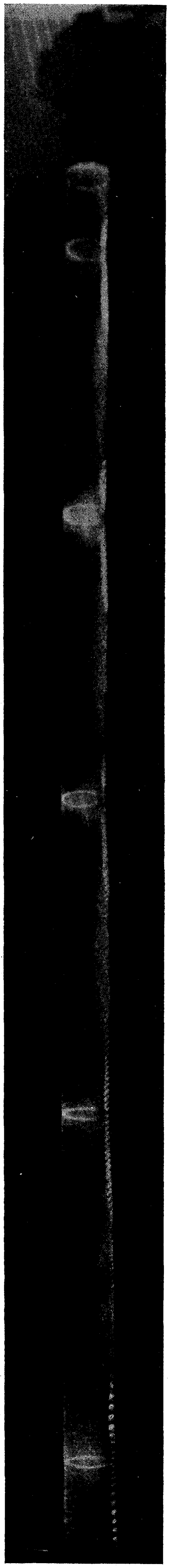


Fig. 14.

Fig. 20.

Fig. 21.

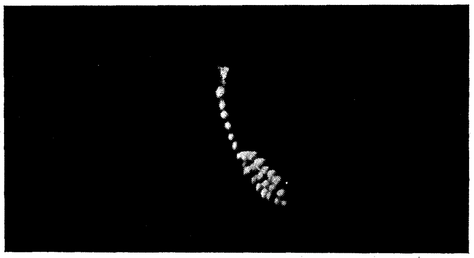


Fig. 22.

Fig. 23.

Fig. 24.

Fig. 25.

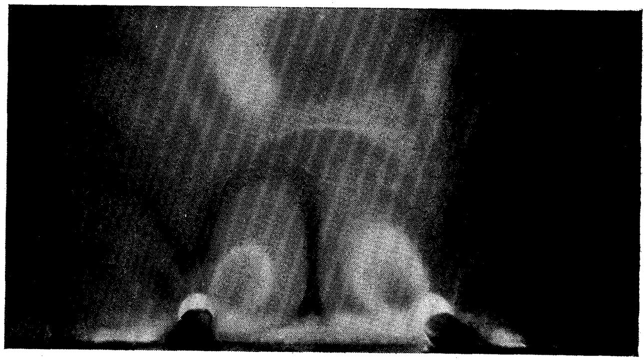
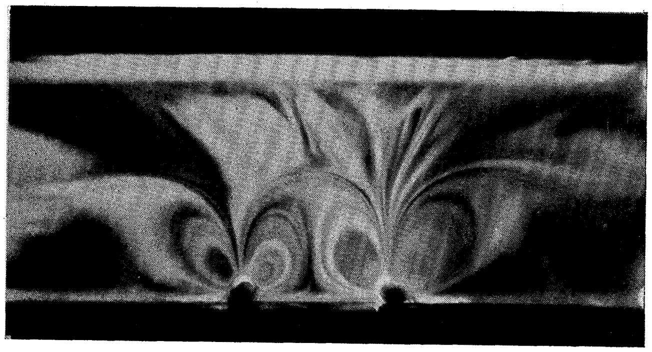


Fig. 26.



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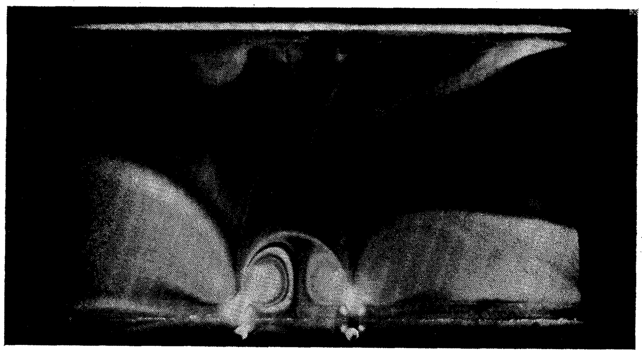

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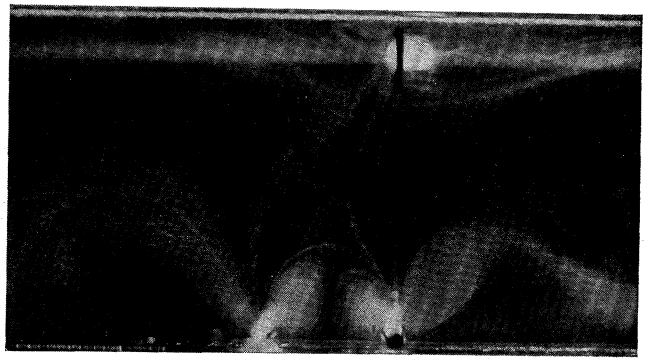

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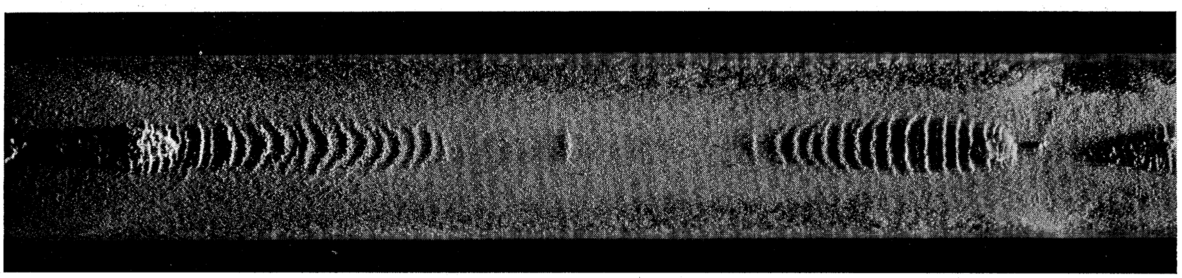
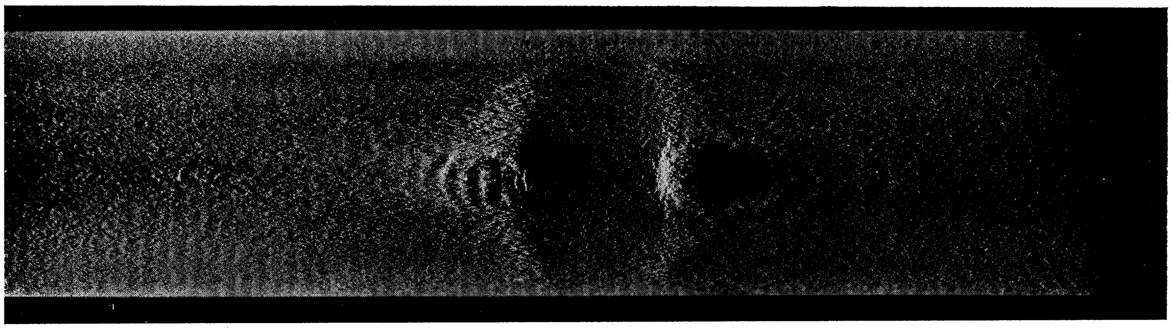


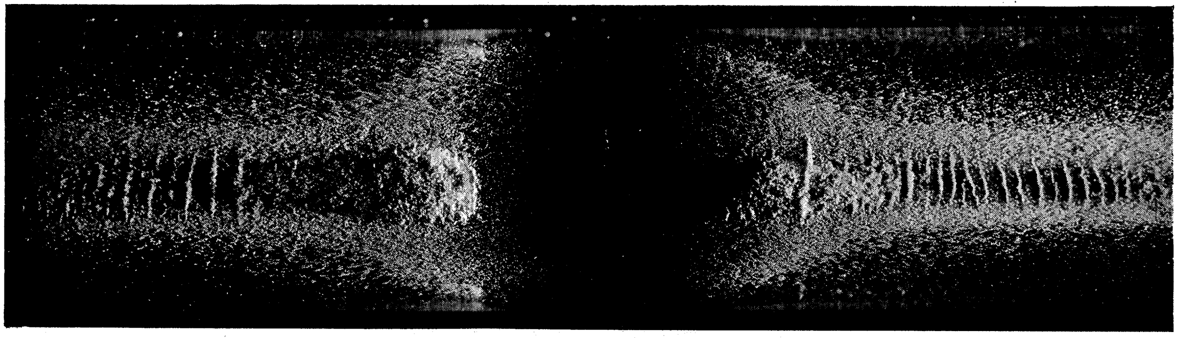
Node. Anti-node. Node.

FIG. 30.



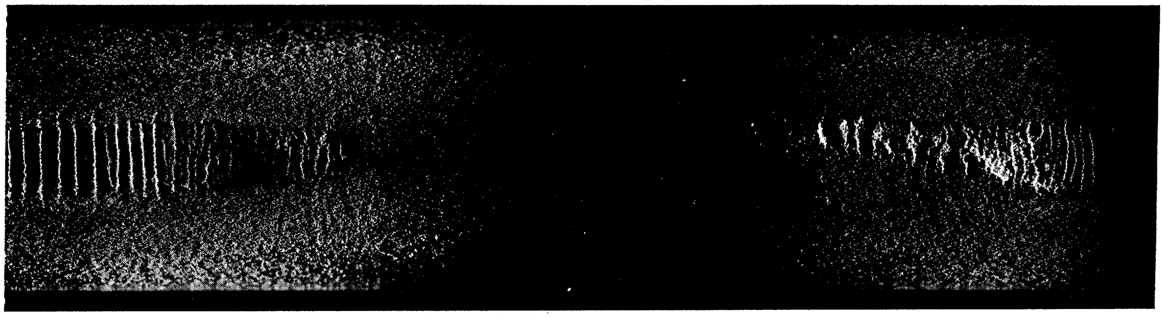
Node.

FIG. 31.



Node.

FIG. 32.



Node.

FIG. 33.

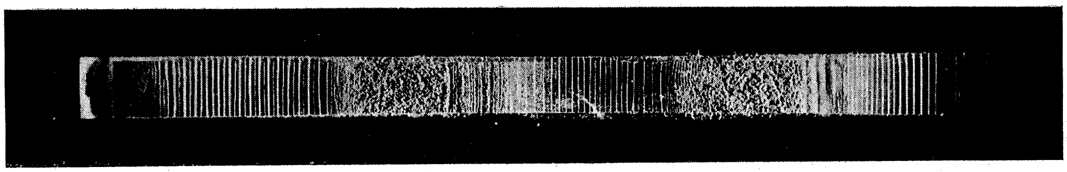


Fig. 34.

Fig. 35.

Fig. 36.

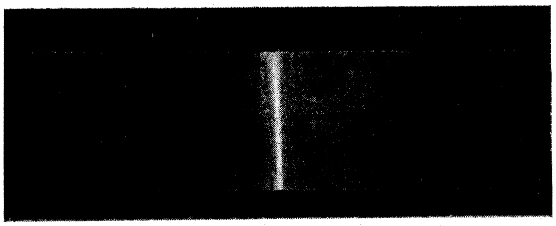


Fig. 37.

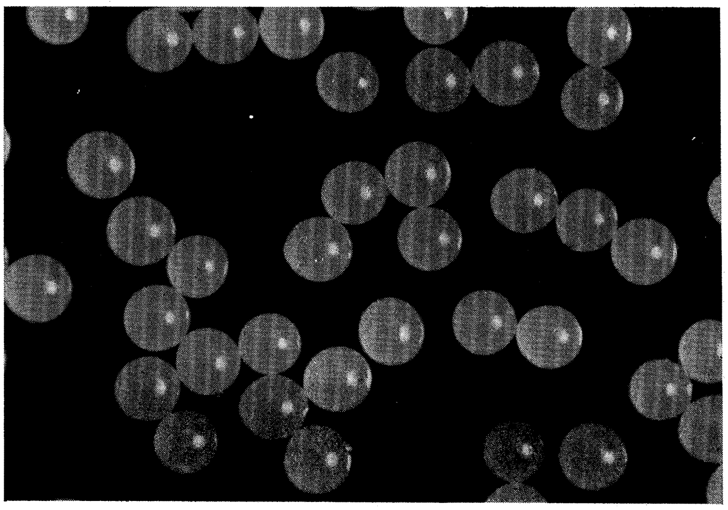


Fig. 38.

Fig. 39.

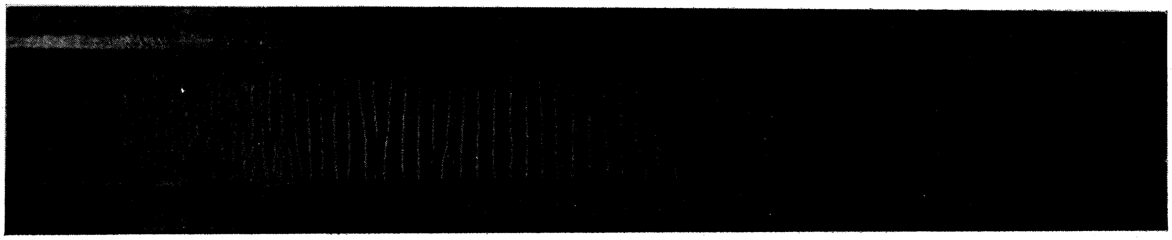


Fig. 40.

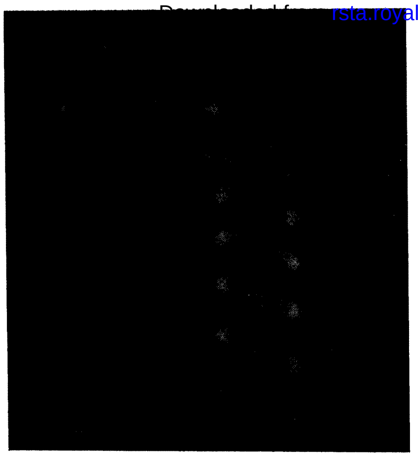


Fig. 41.

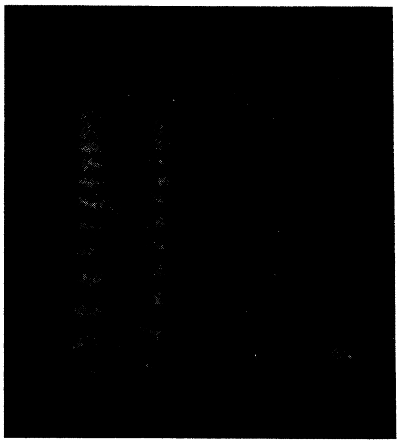
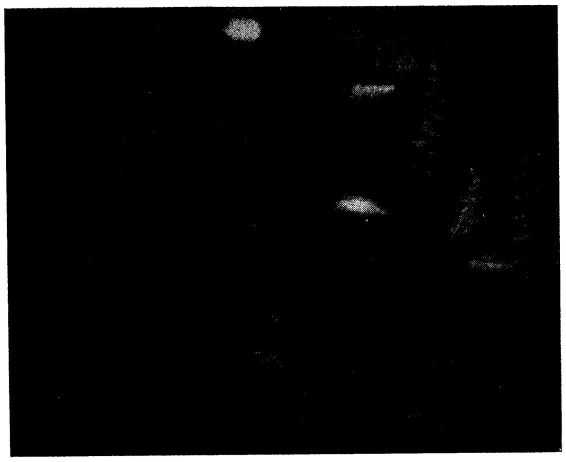


Fig. 42.



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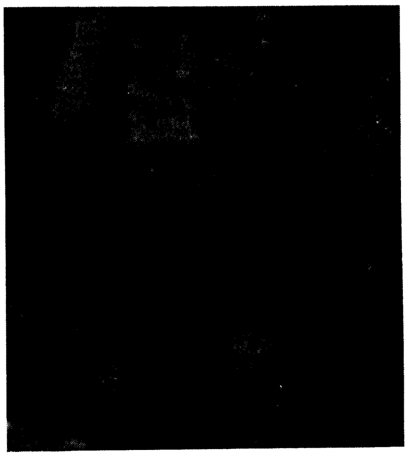

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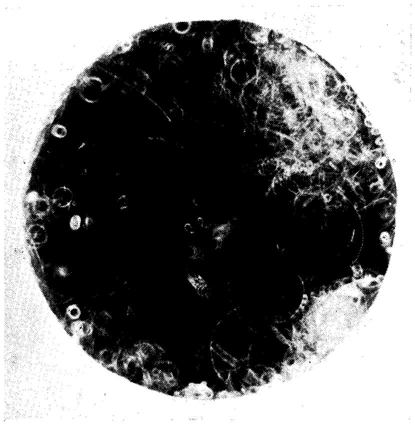


Fig. 45.

Fig. 46.

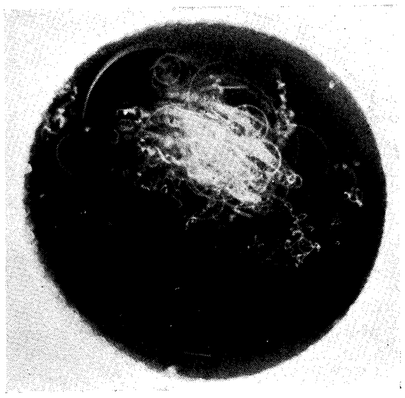


Fig. 47.

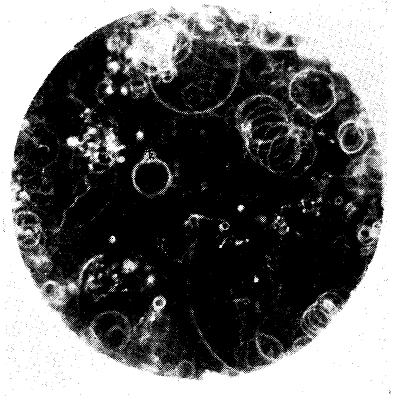


Fig. 48.

FIG. 5. × 50.

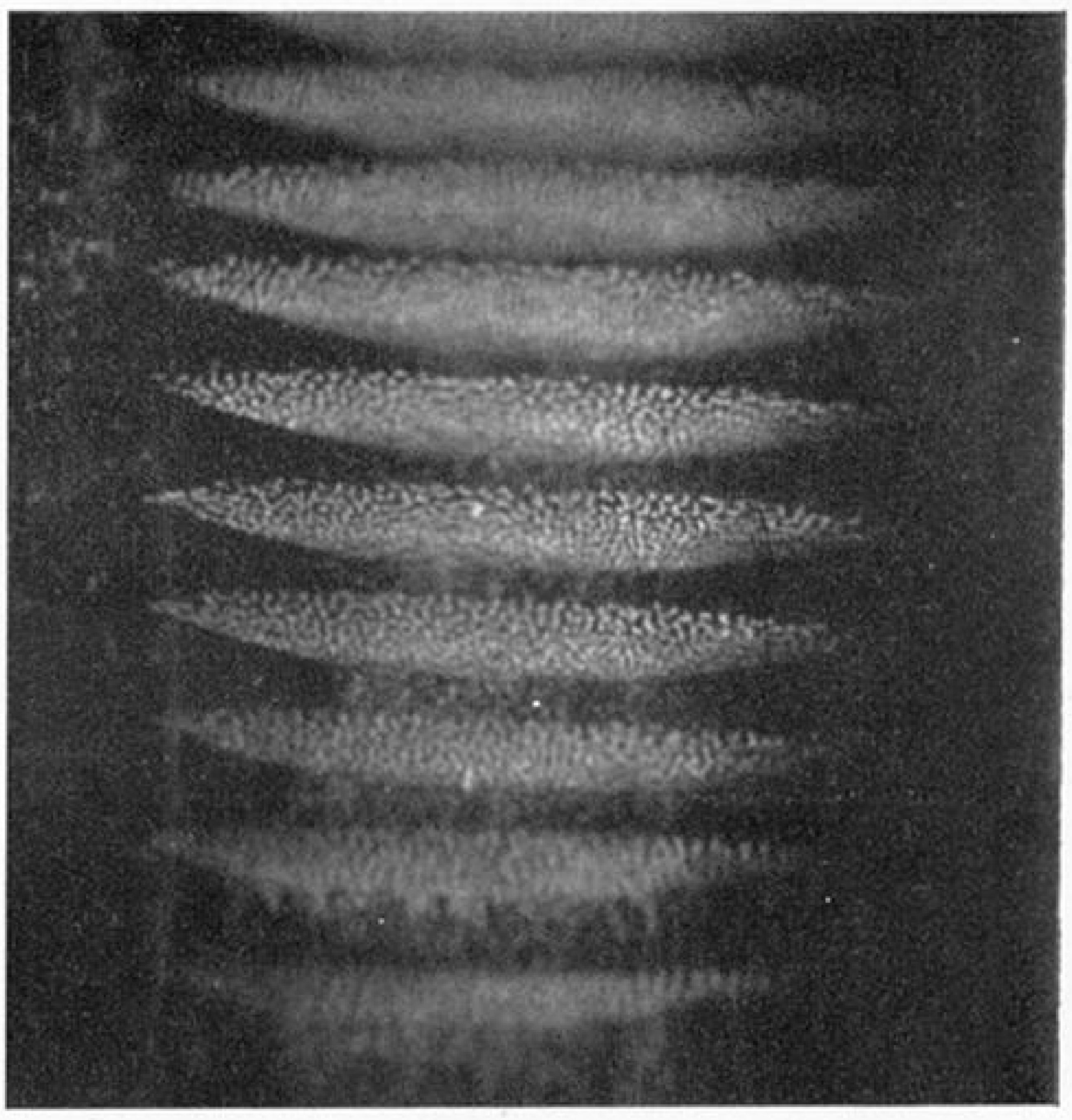


FIG. 6.

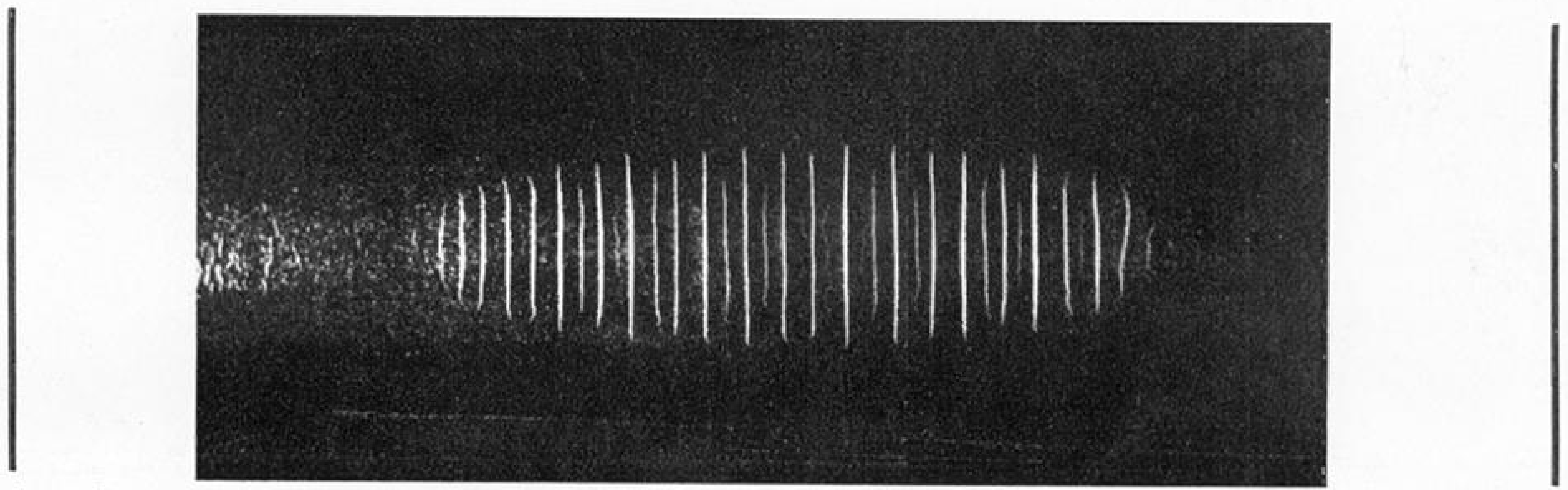


FIG. 7.

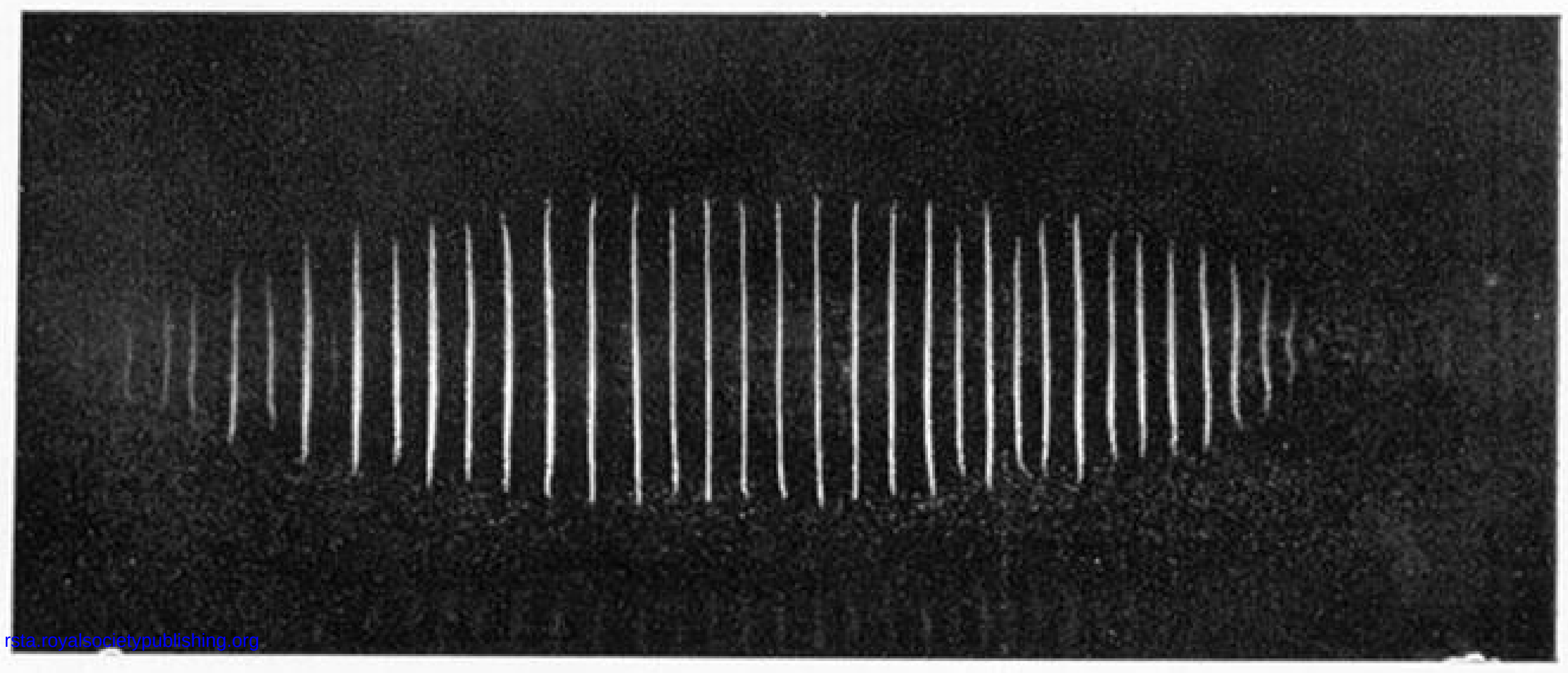


FIG. 8.

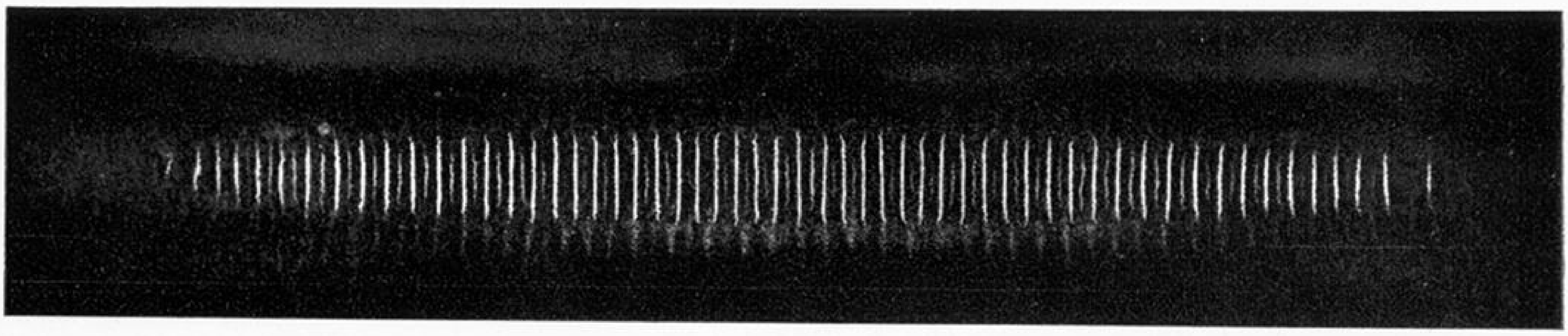


FIG. 9.

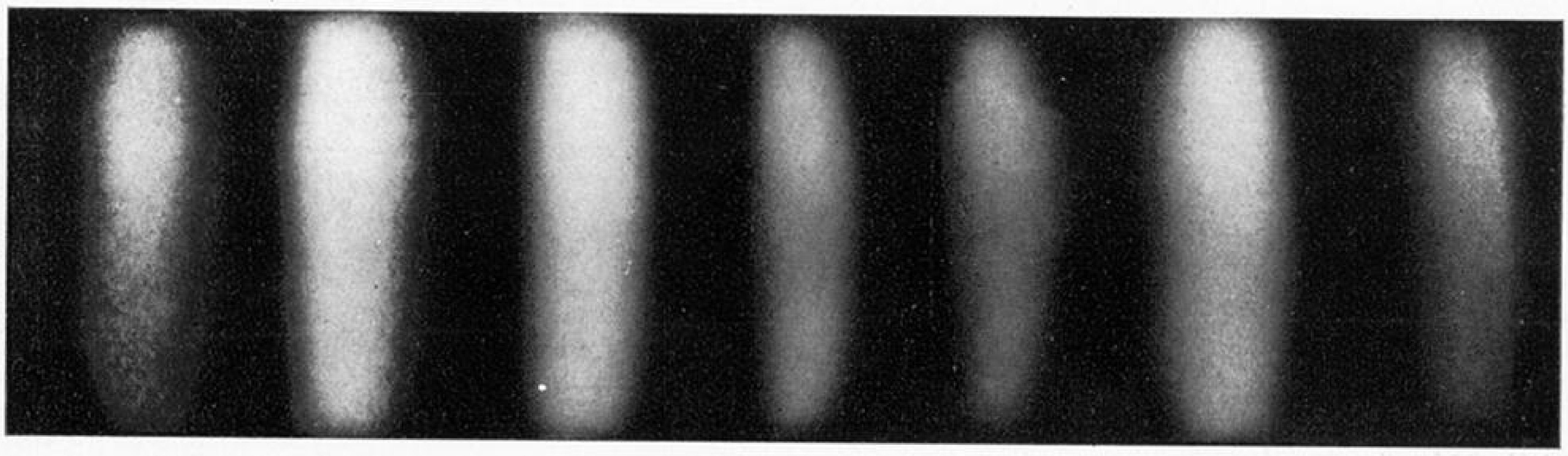


FIG. 10.

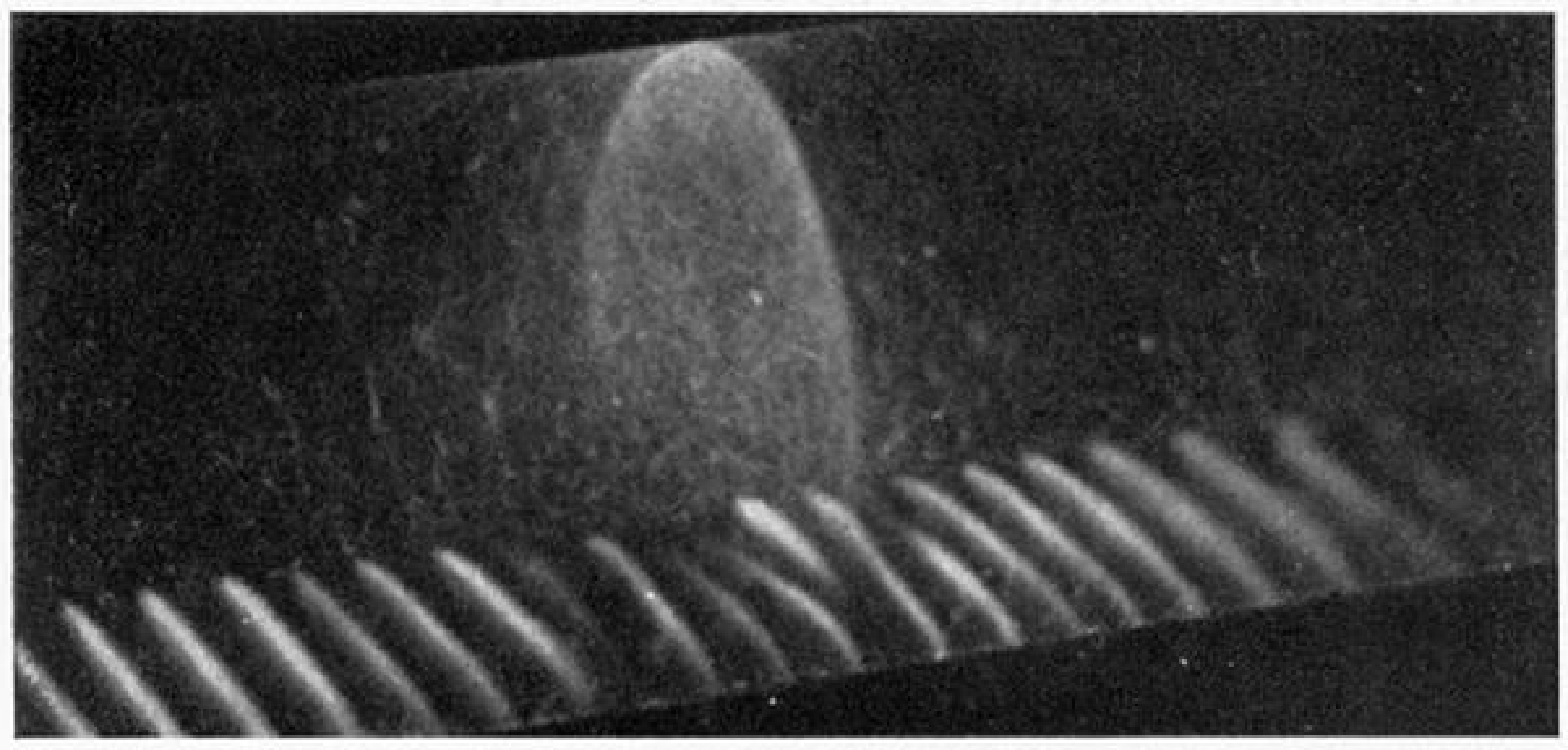
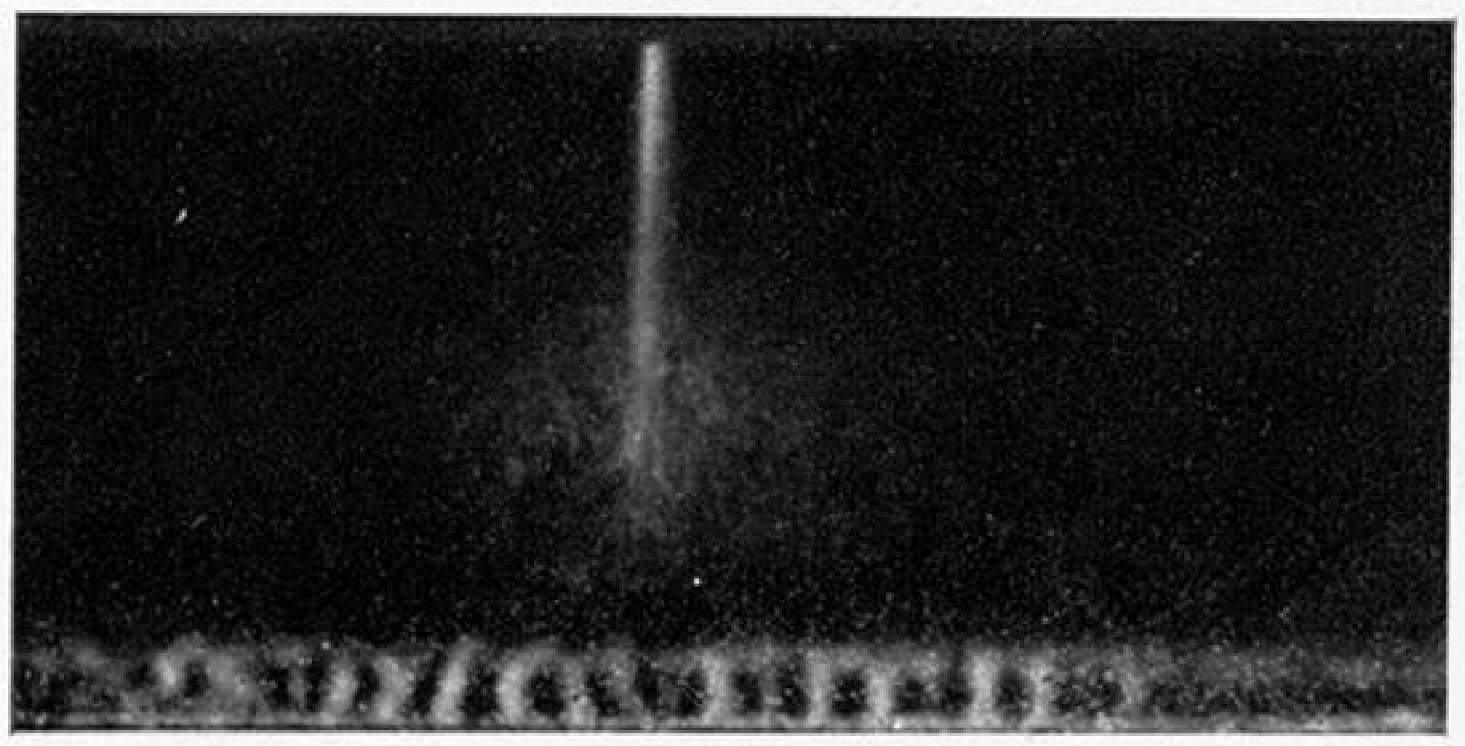


Fig. 11.



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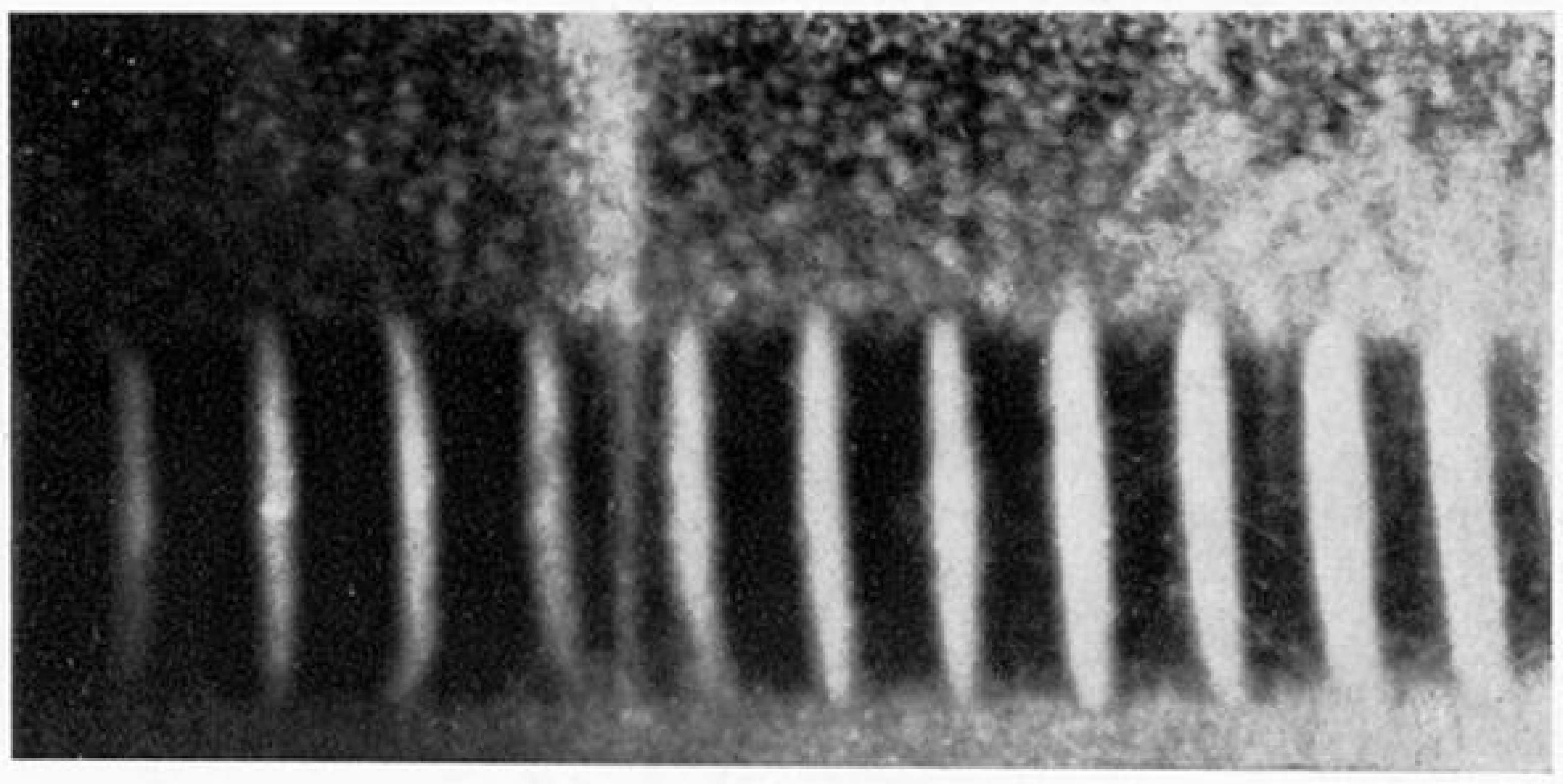
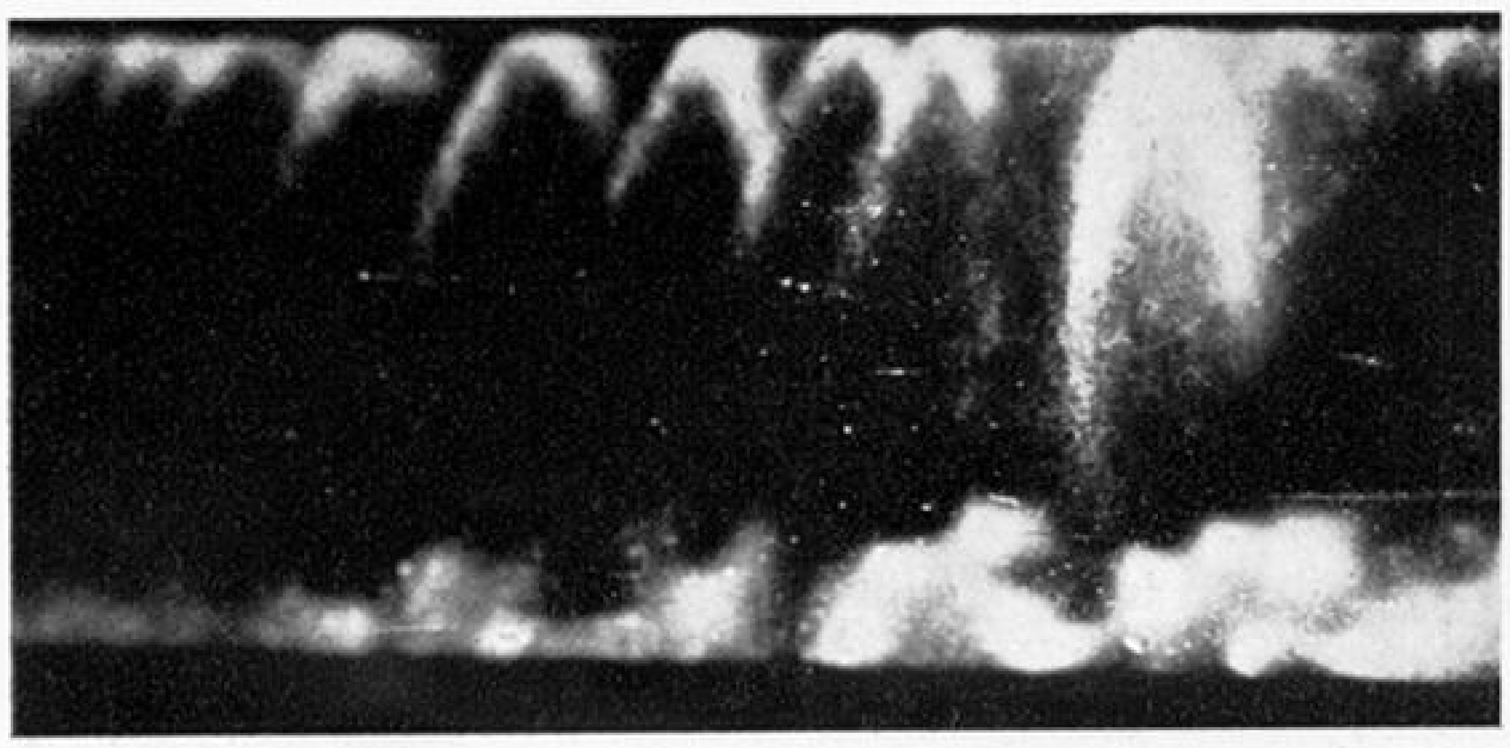


Fig. 13.



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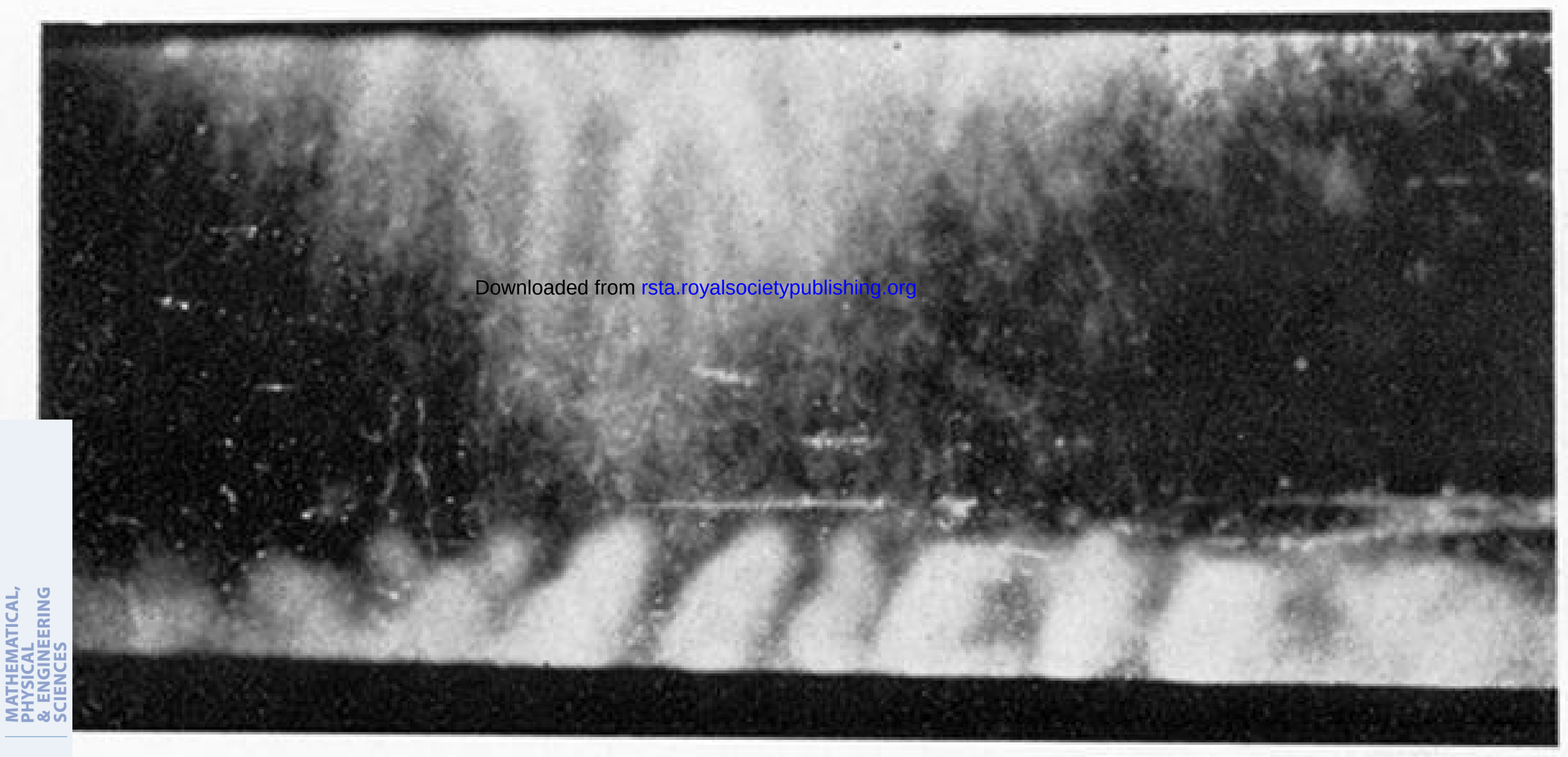


Fig. 16.

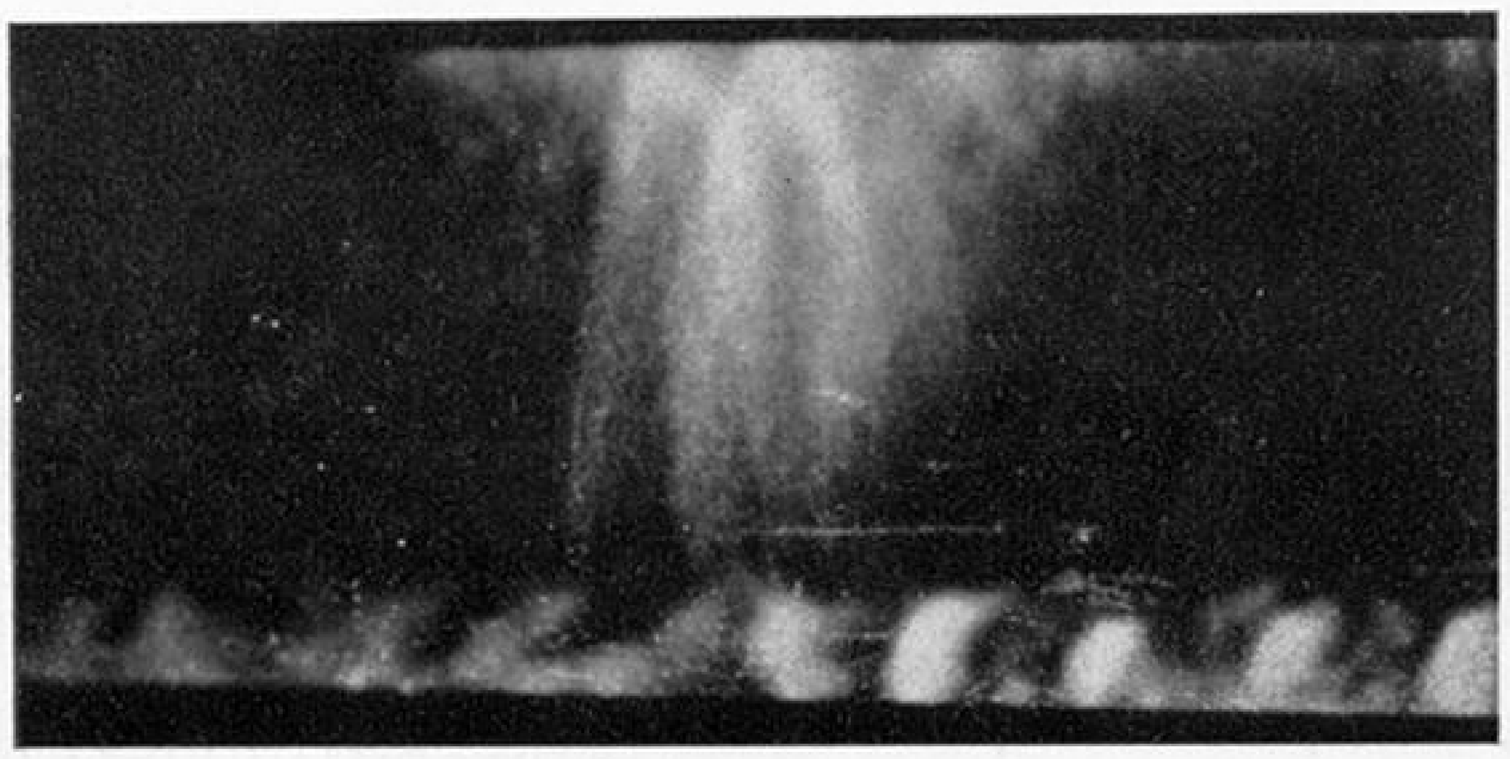


Fig. 17.

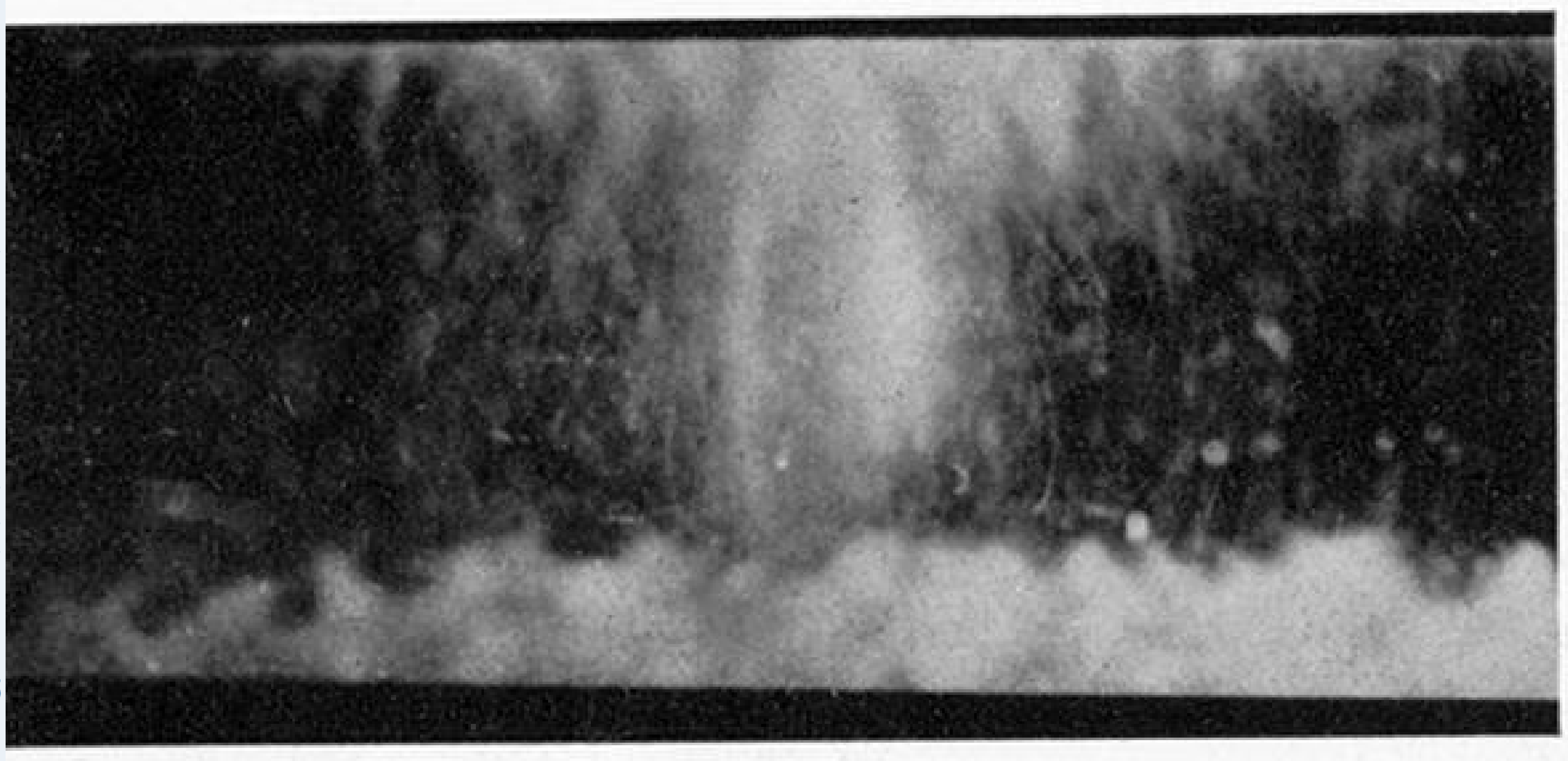


Fig. 18.

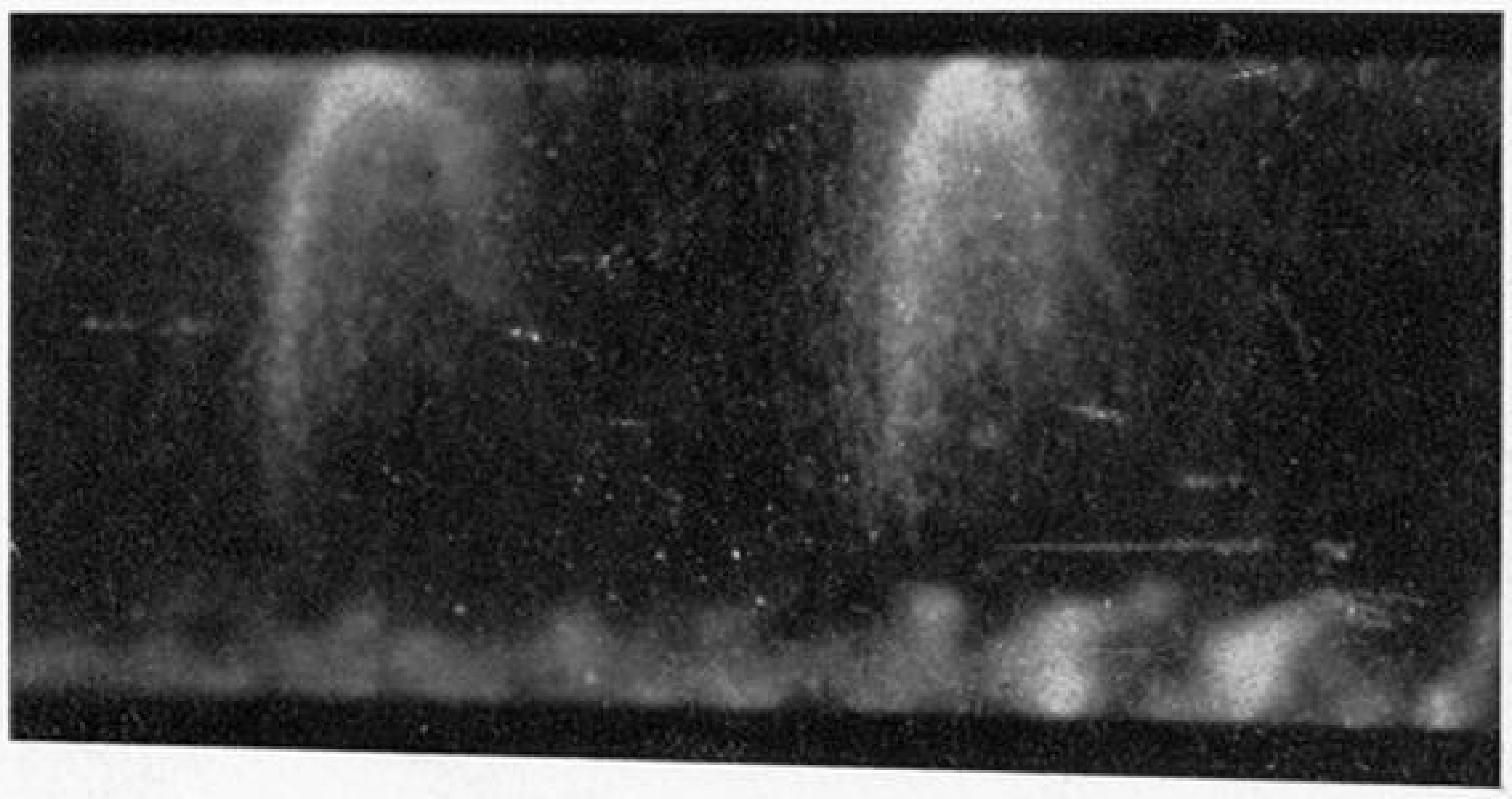


Fig. 19.

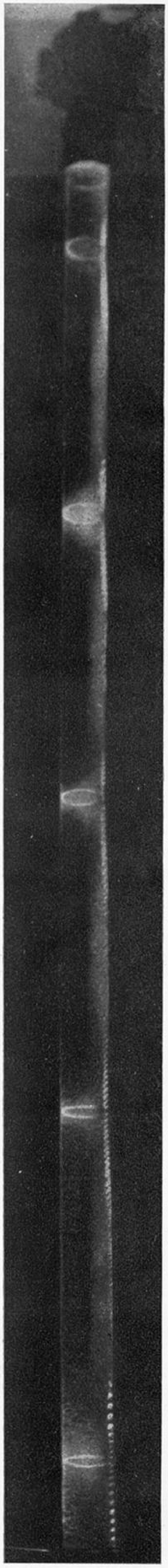


Fig. 14.

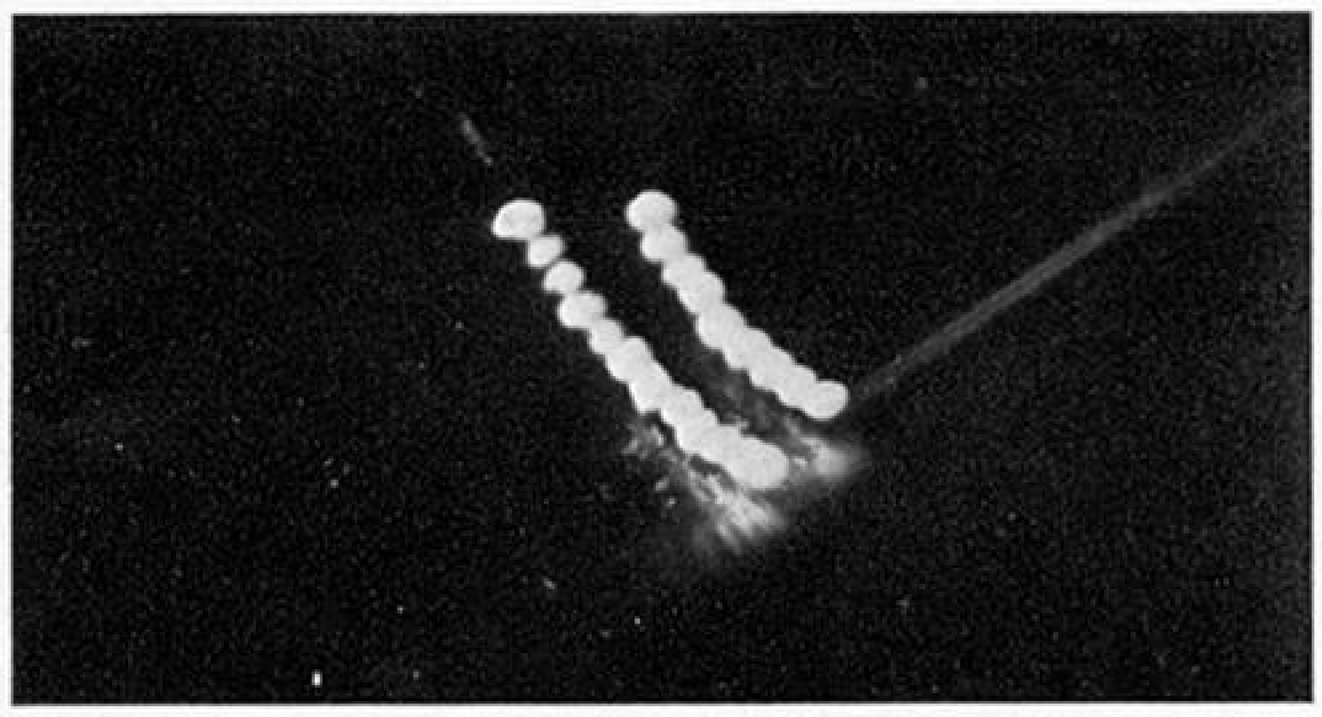


FIG. 20.

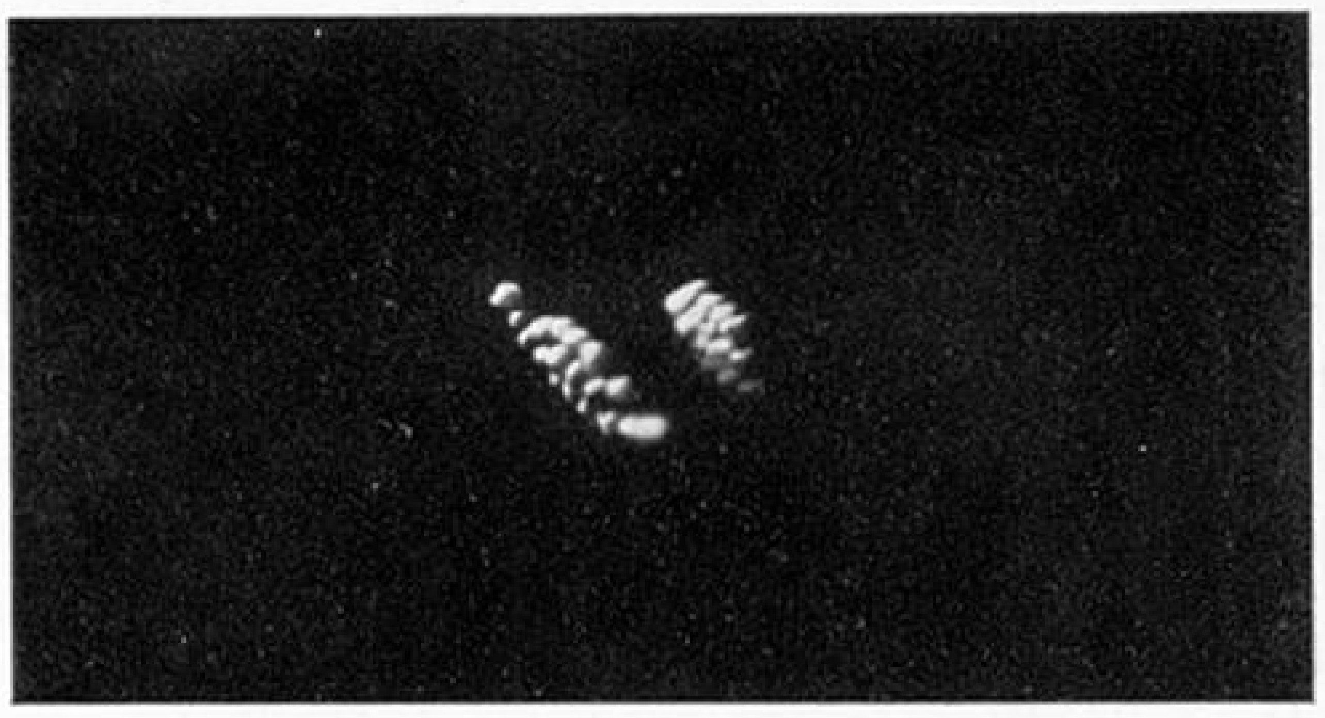


FIG. 21.

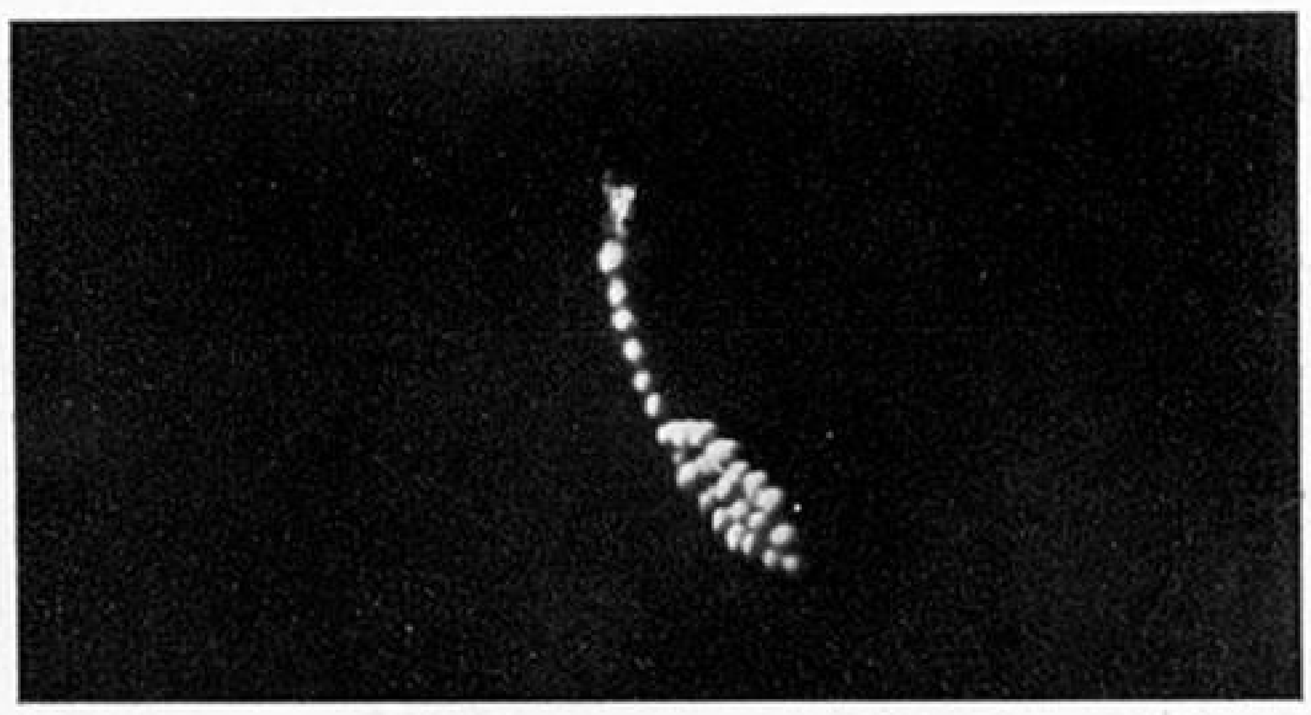


FIG. 22.

FIG. 23.

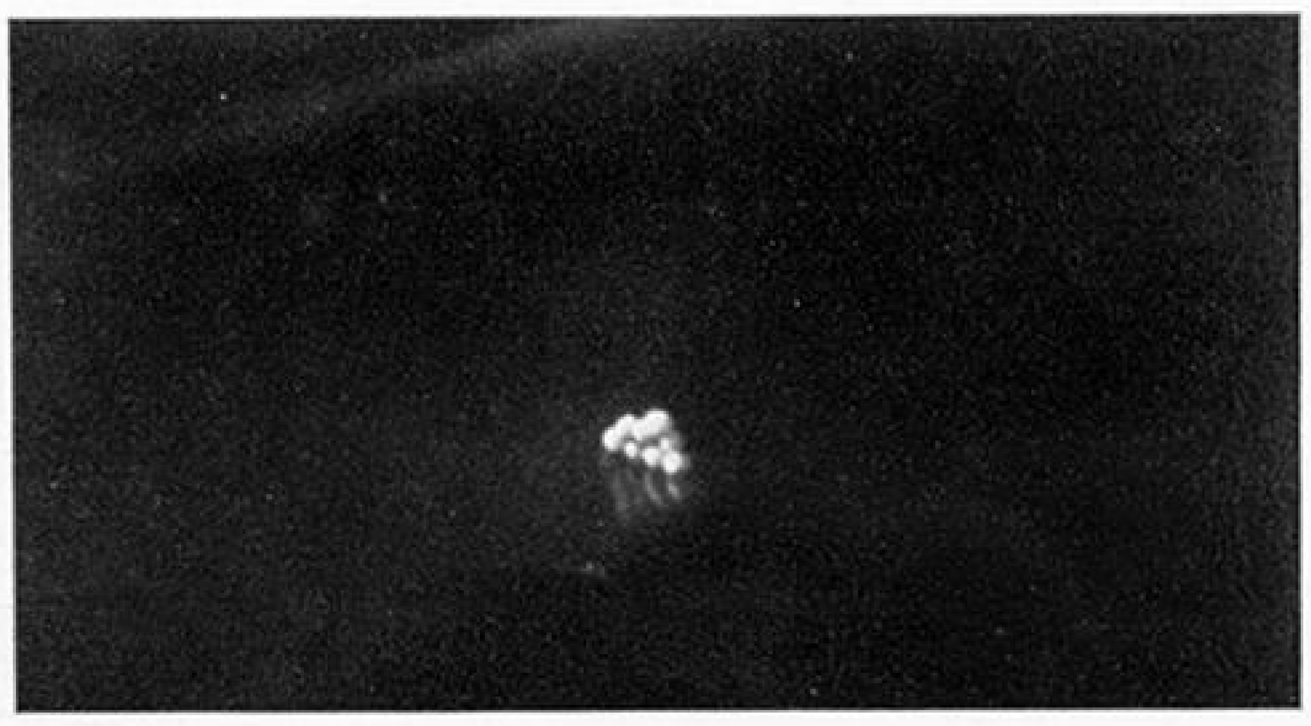


FIG. 24.

FIG. 25.

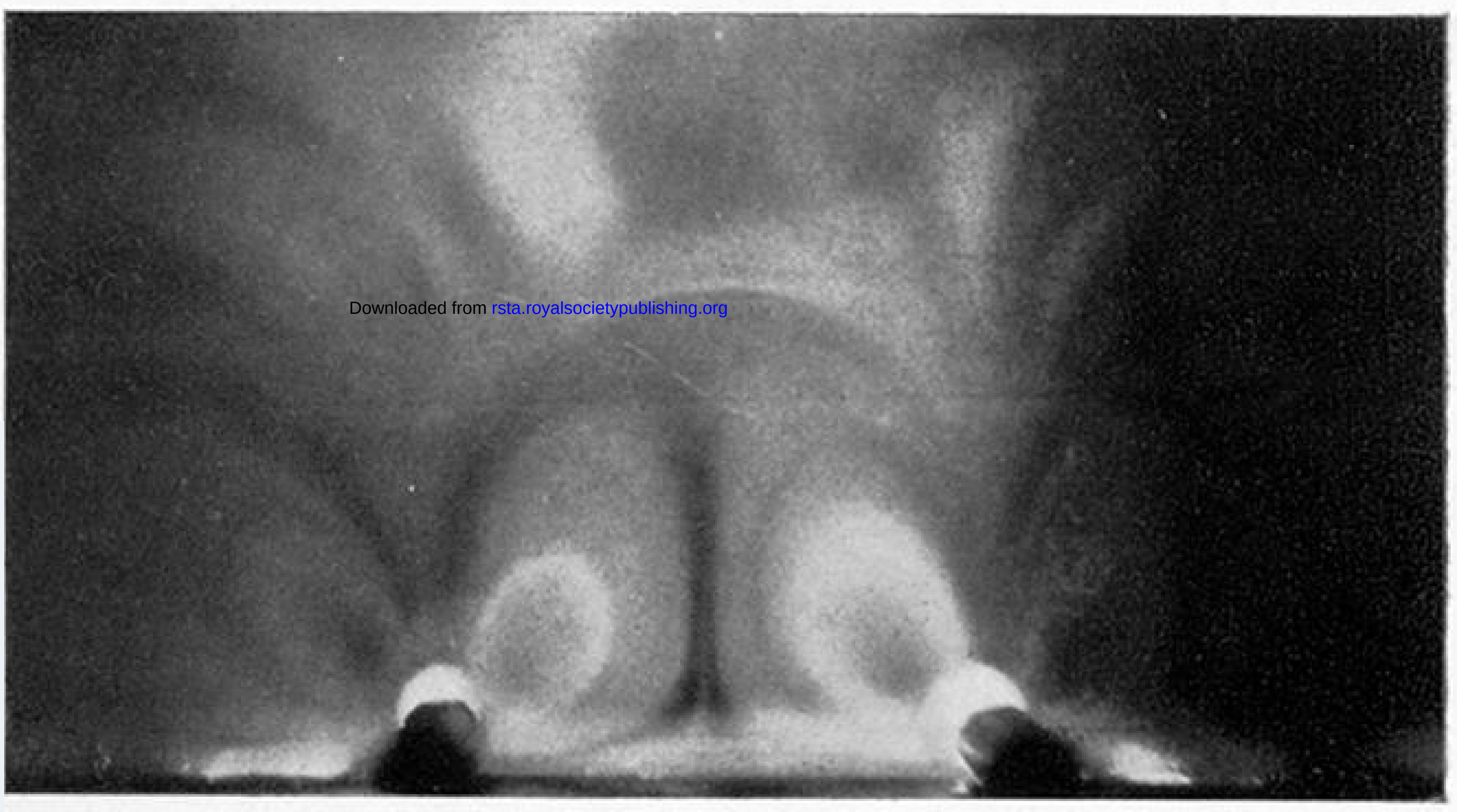


FIG. 26.

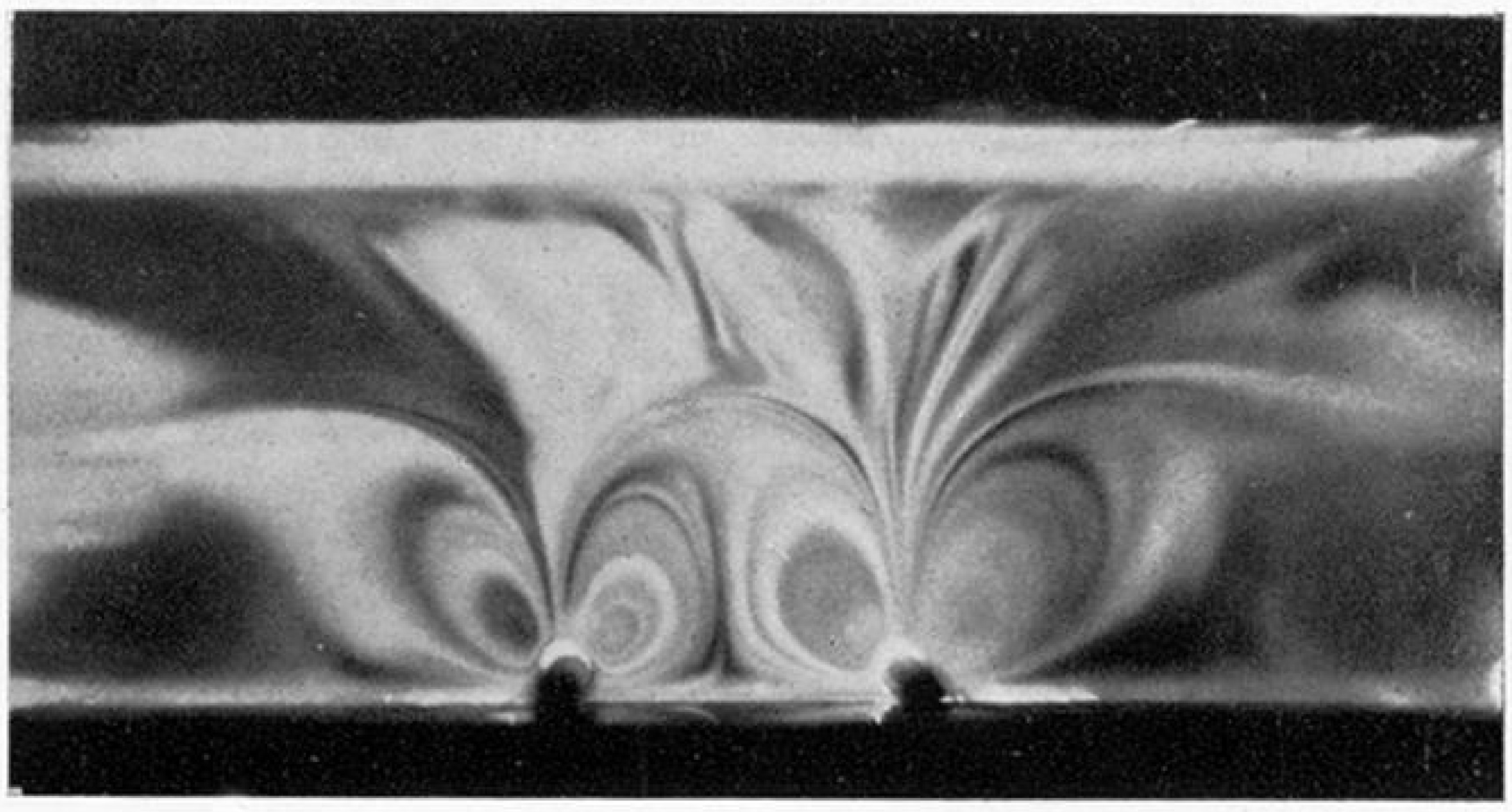


FIG. 27.

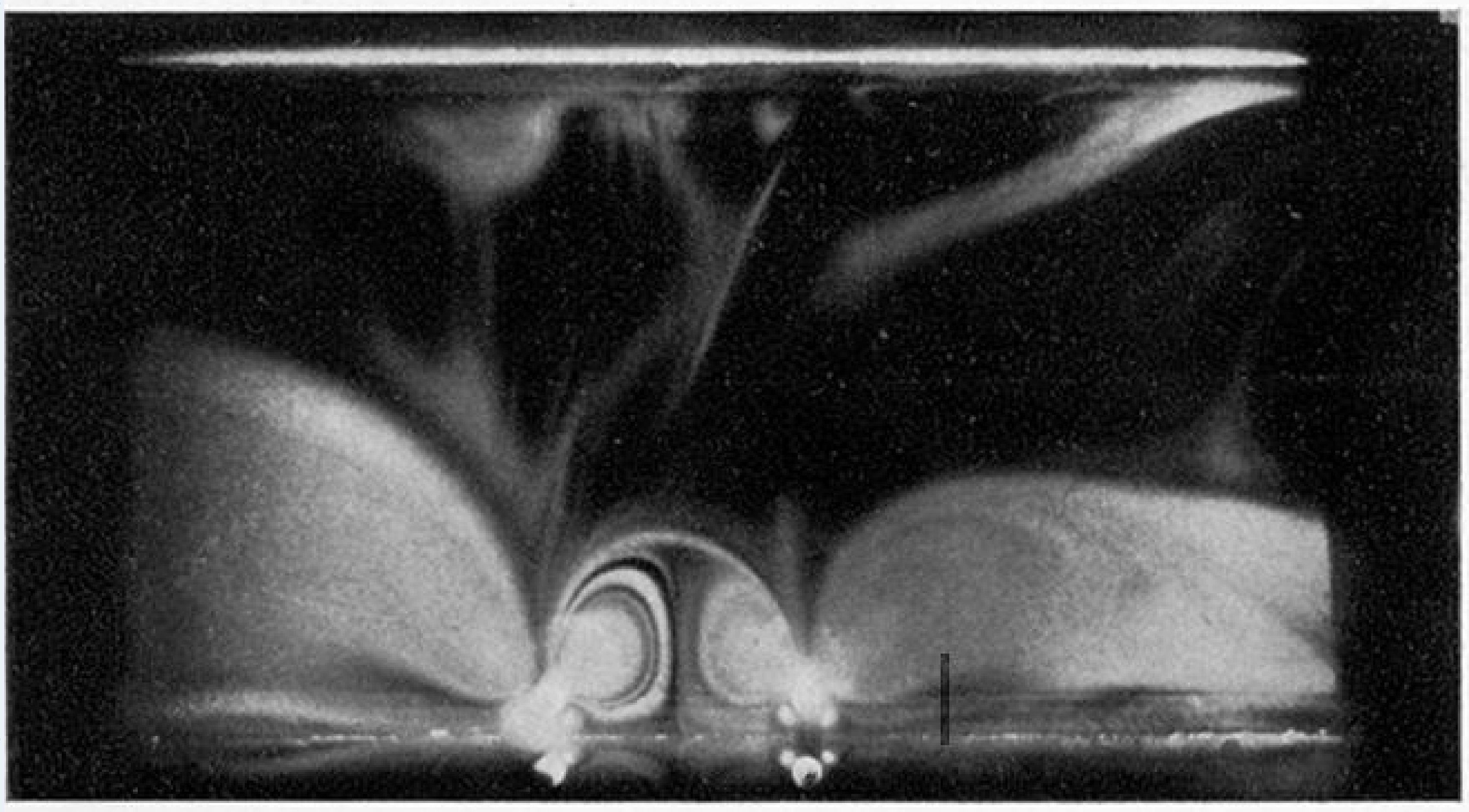


FIG. 28.

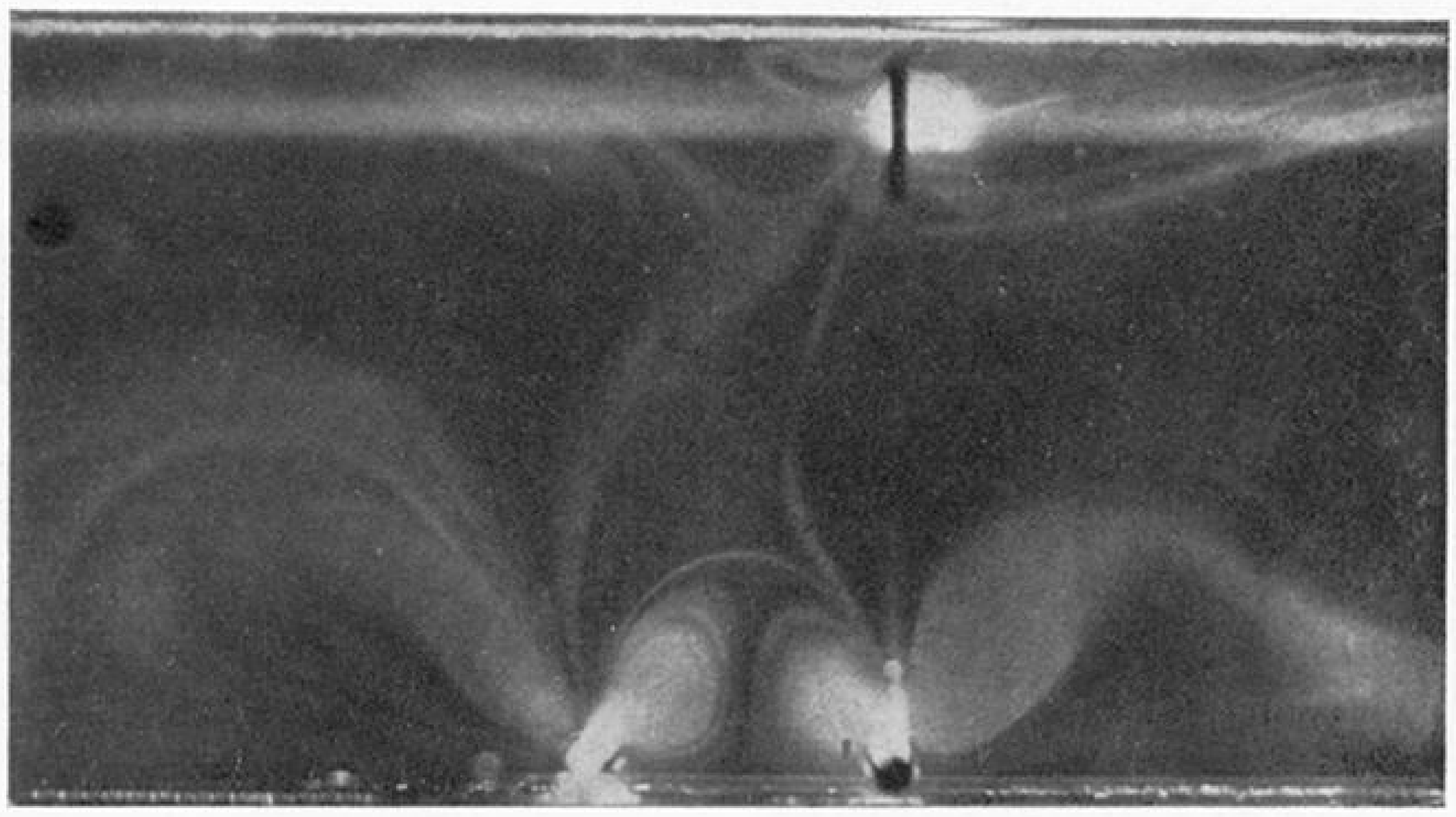
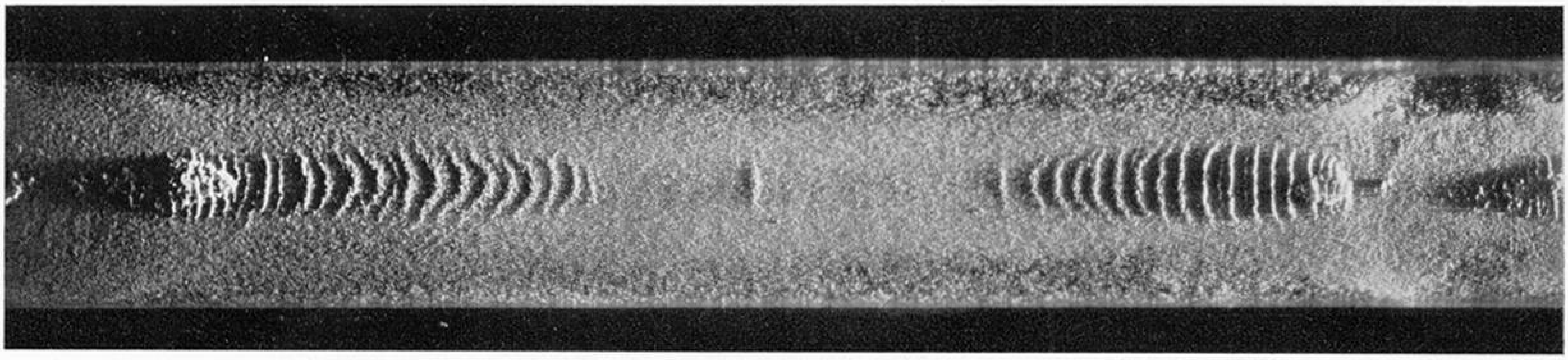
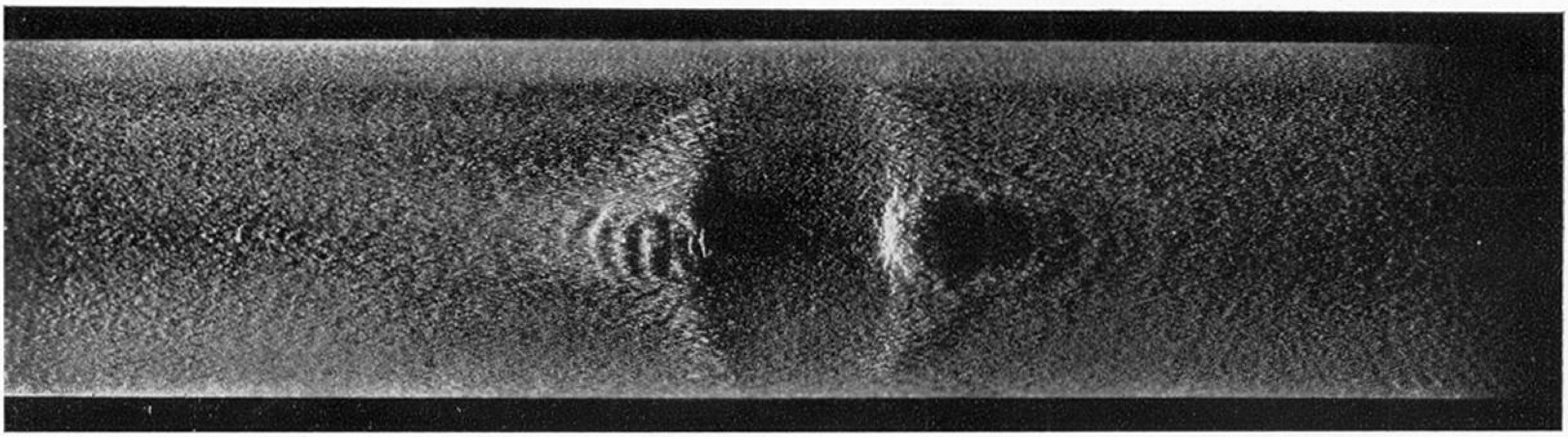


FIG. 29.



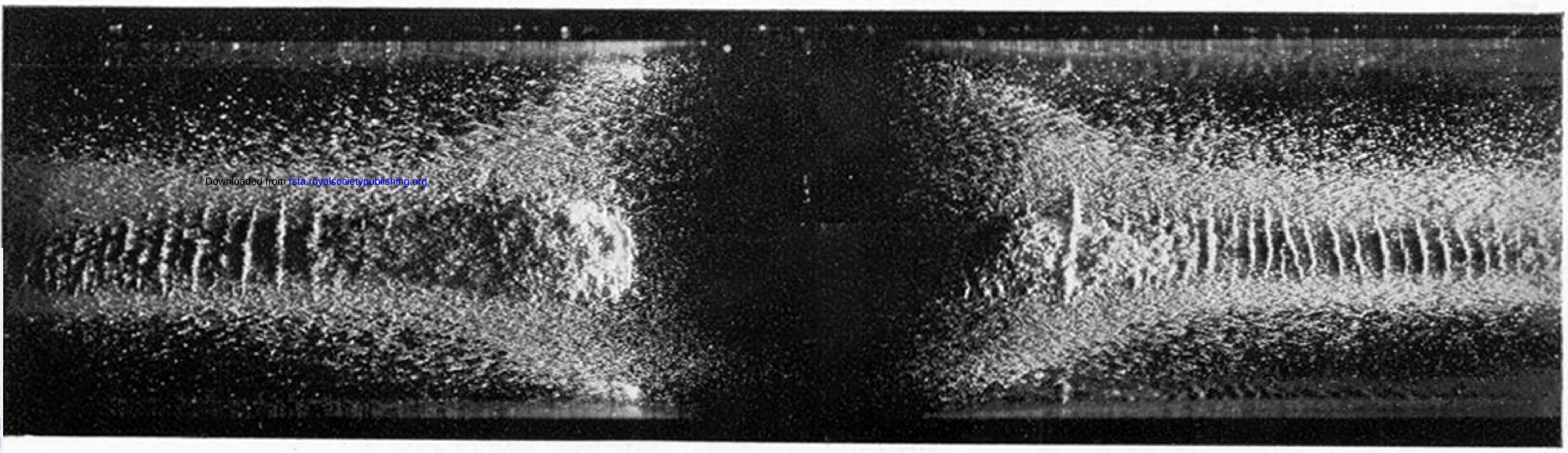
Node. Anti-node. Node.

FIG. 30.



Node.

FIG. 31.



Node.

FIG. 32.

Node.

FIG. 33.

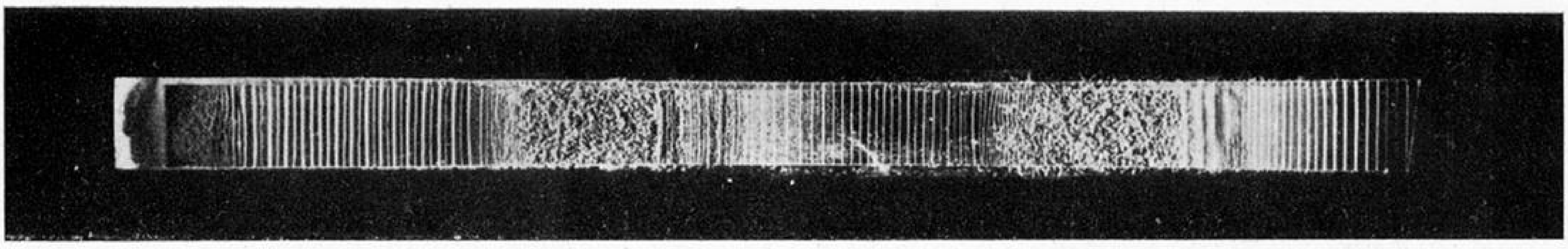


Fig. 34.

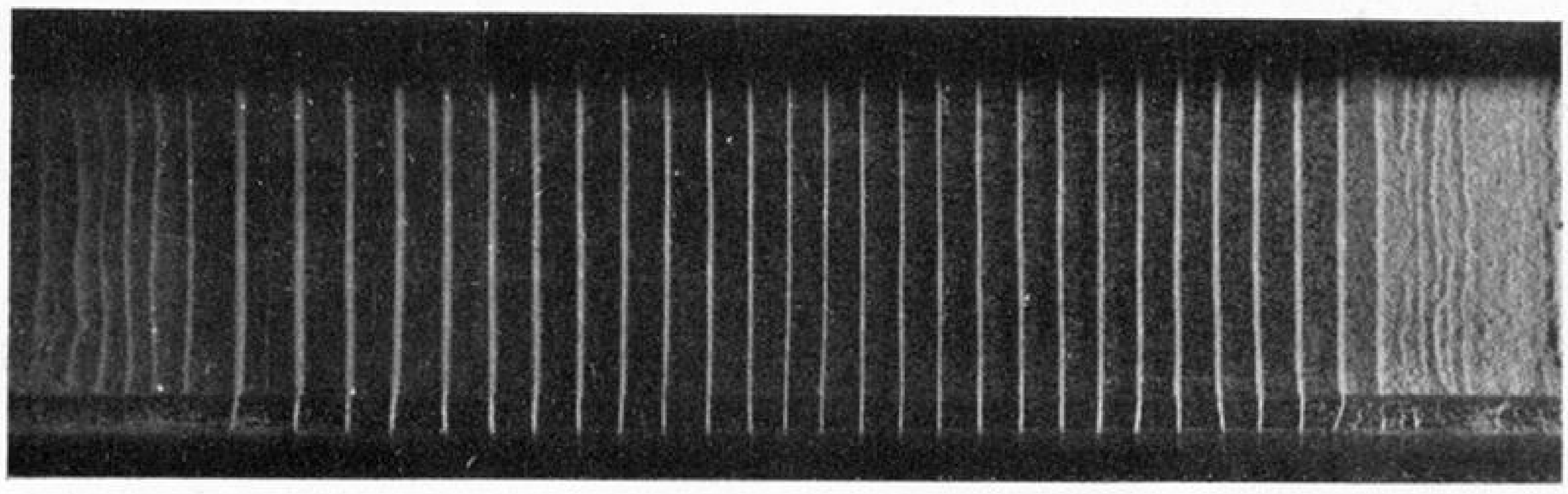


Fig. 35.

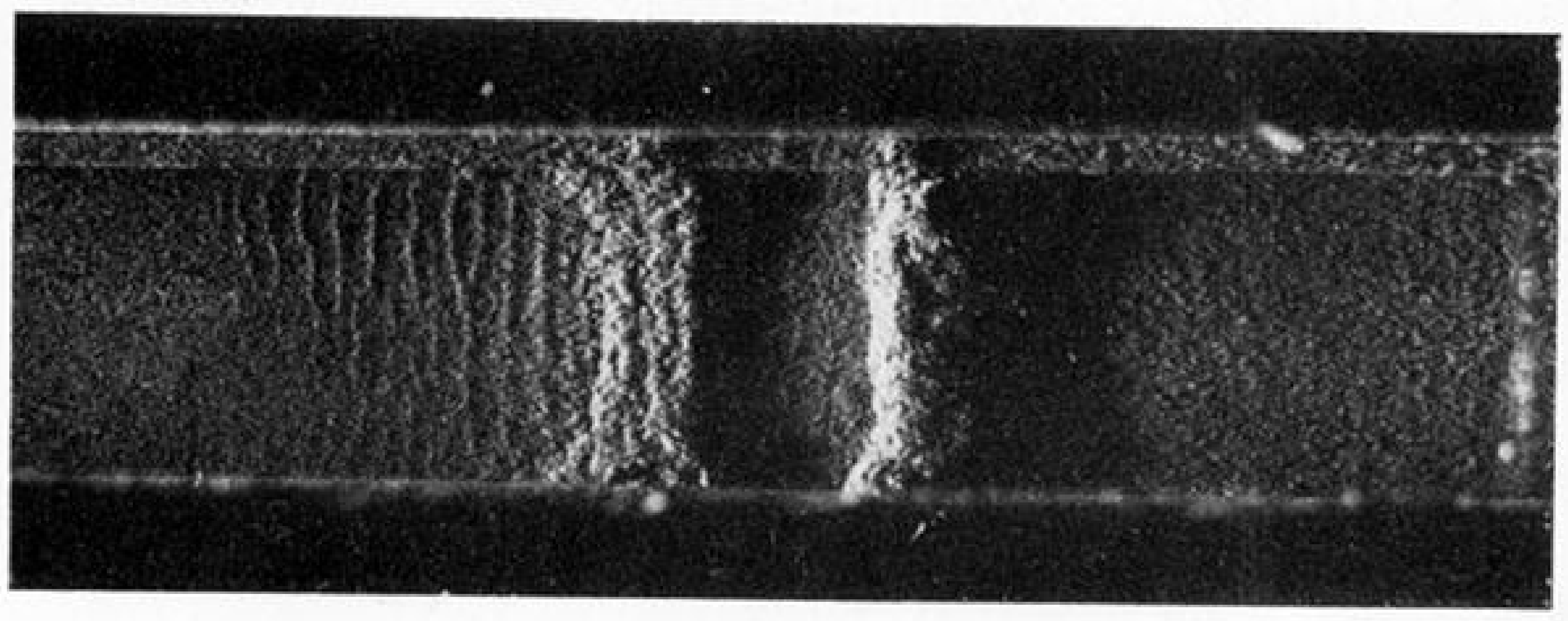


Fig. 36.

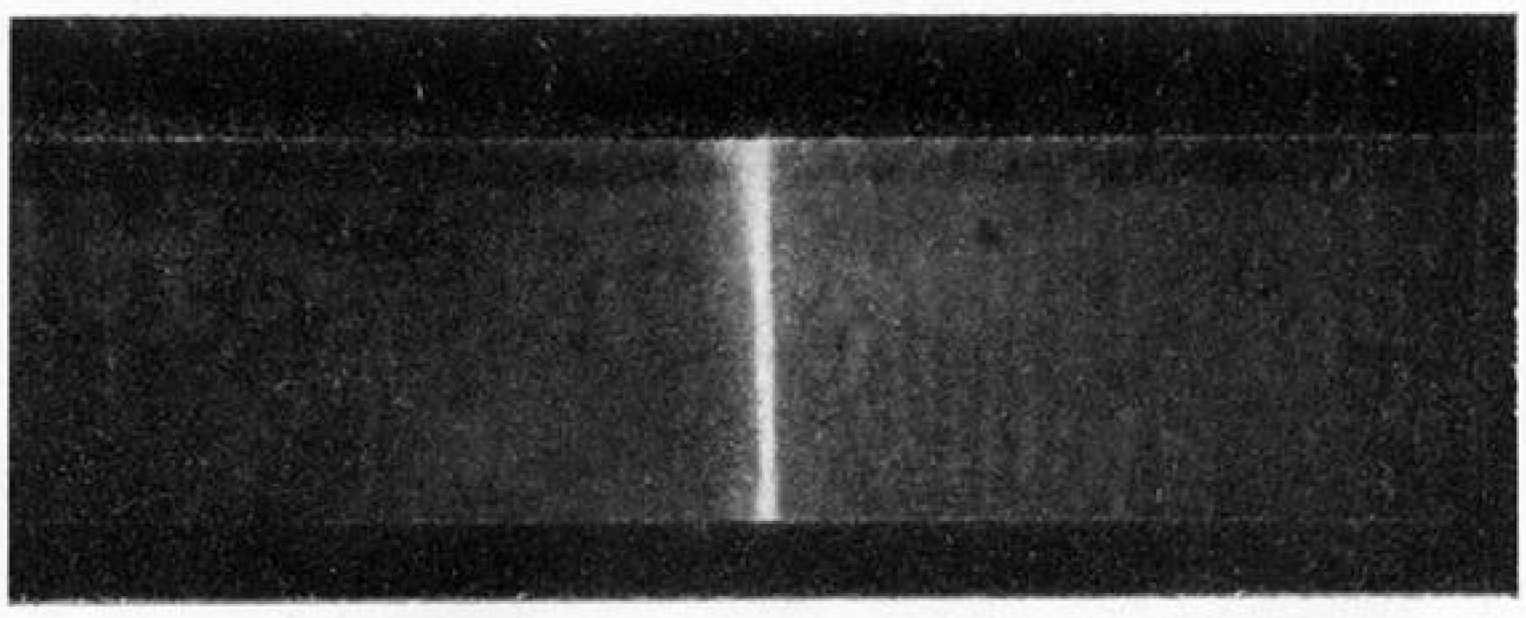


Fig. 37.

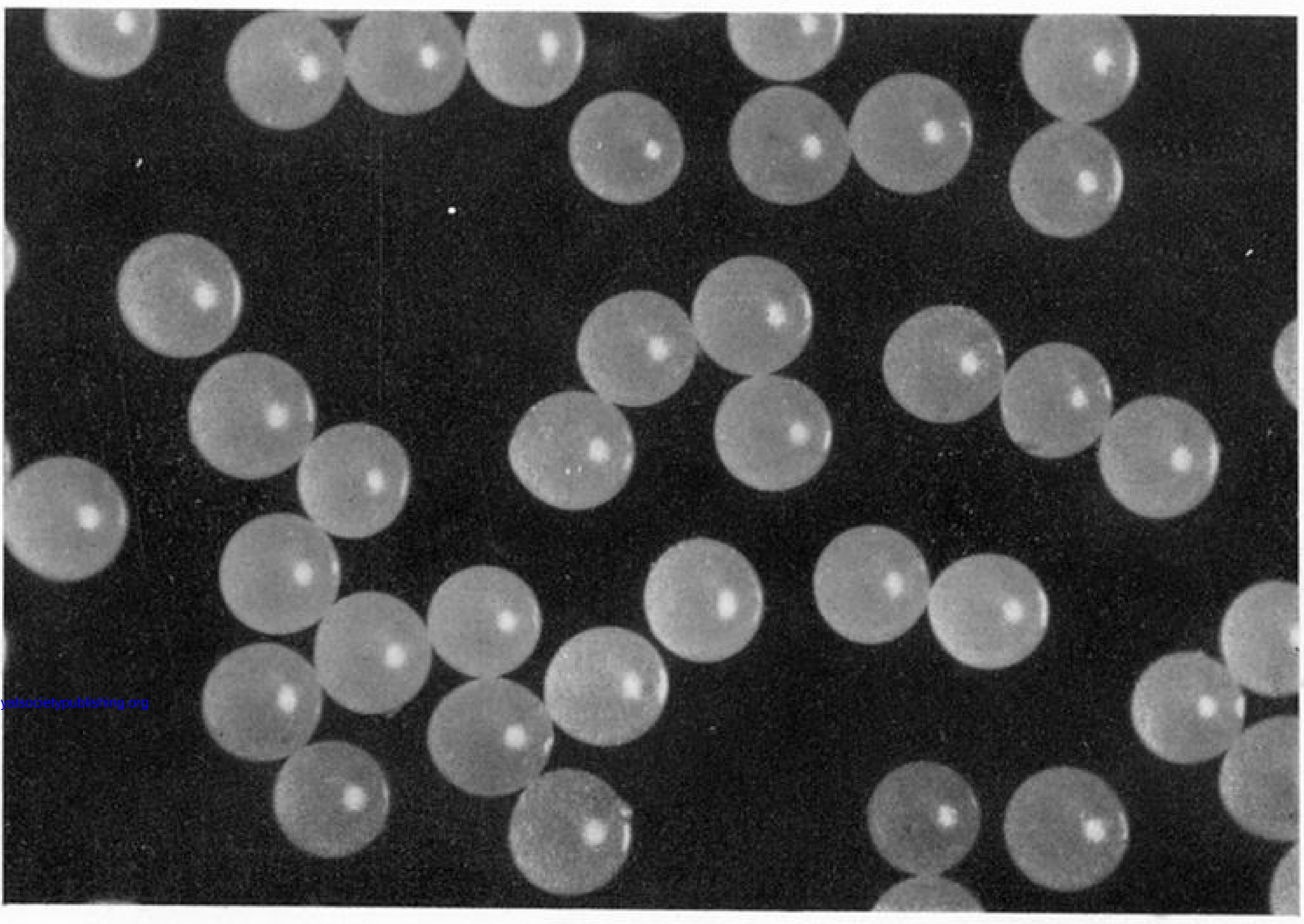


Fig. 38.

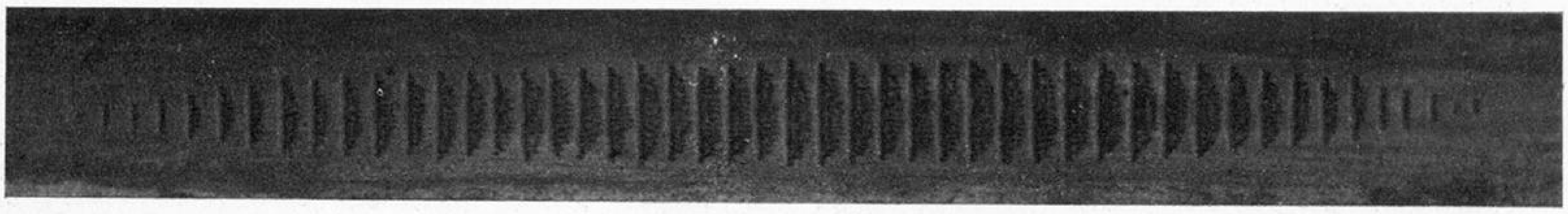


Fig. 39.

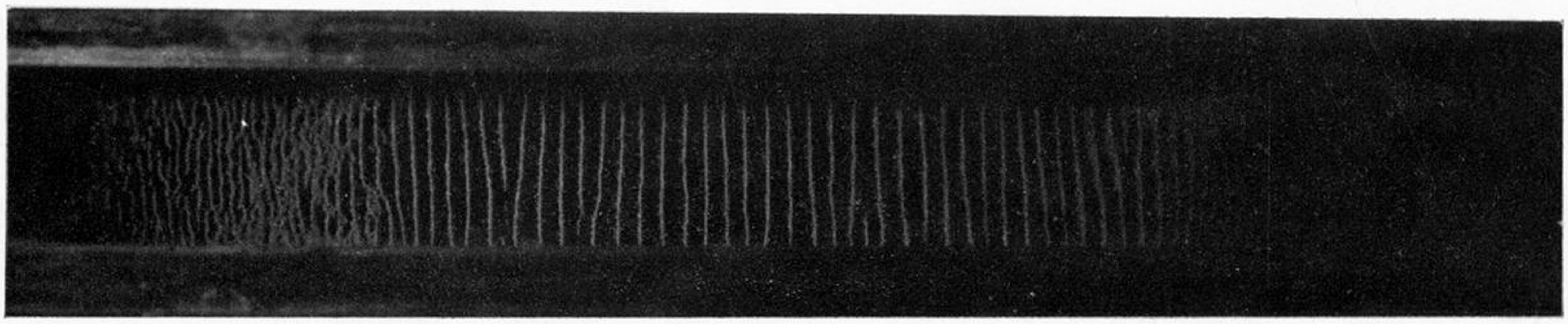


Fig. 40.

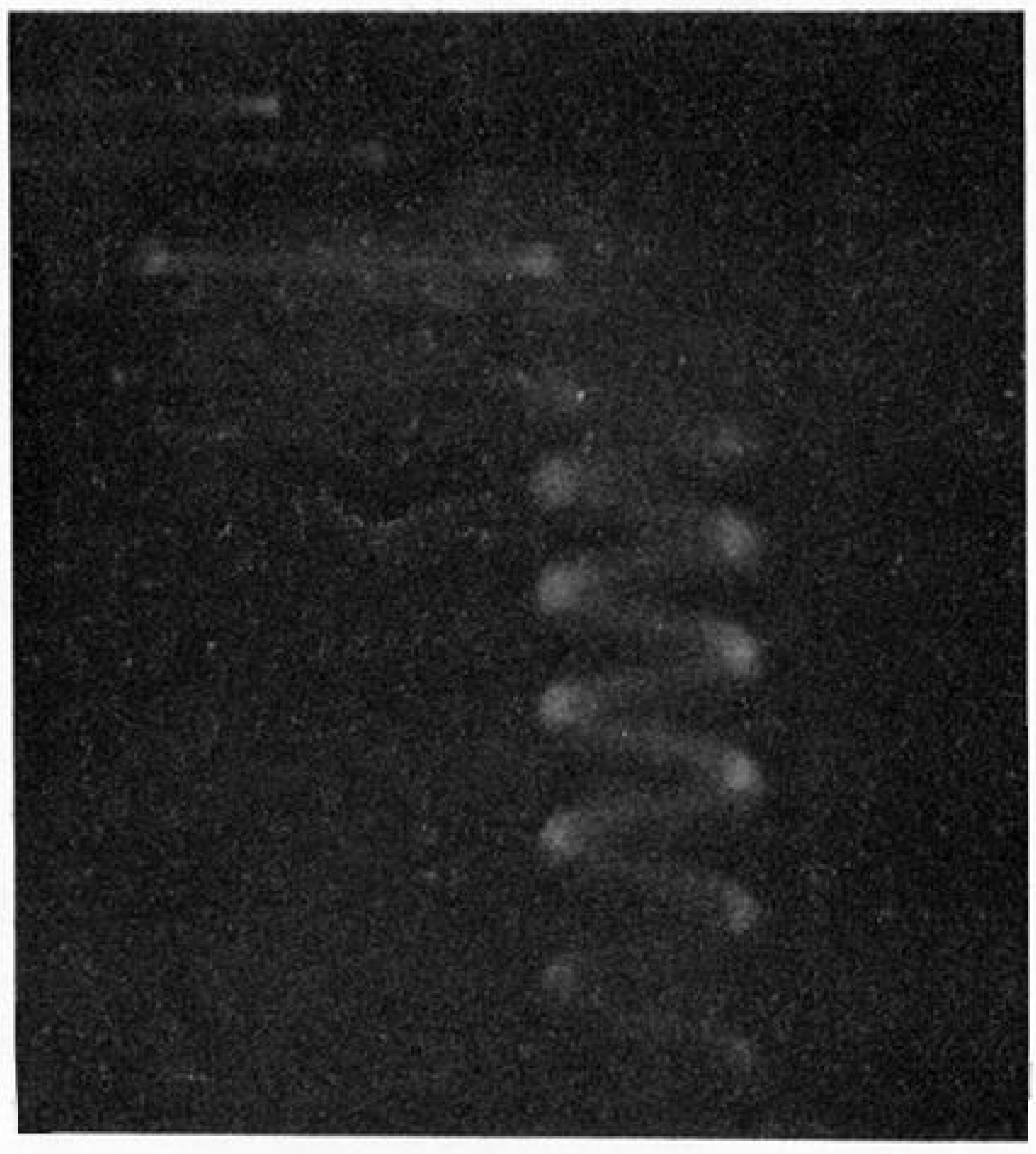
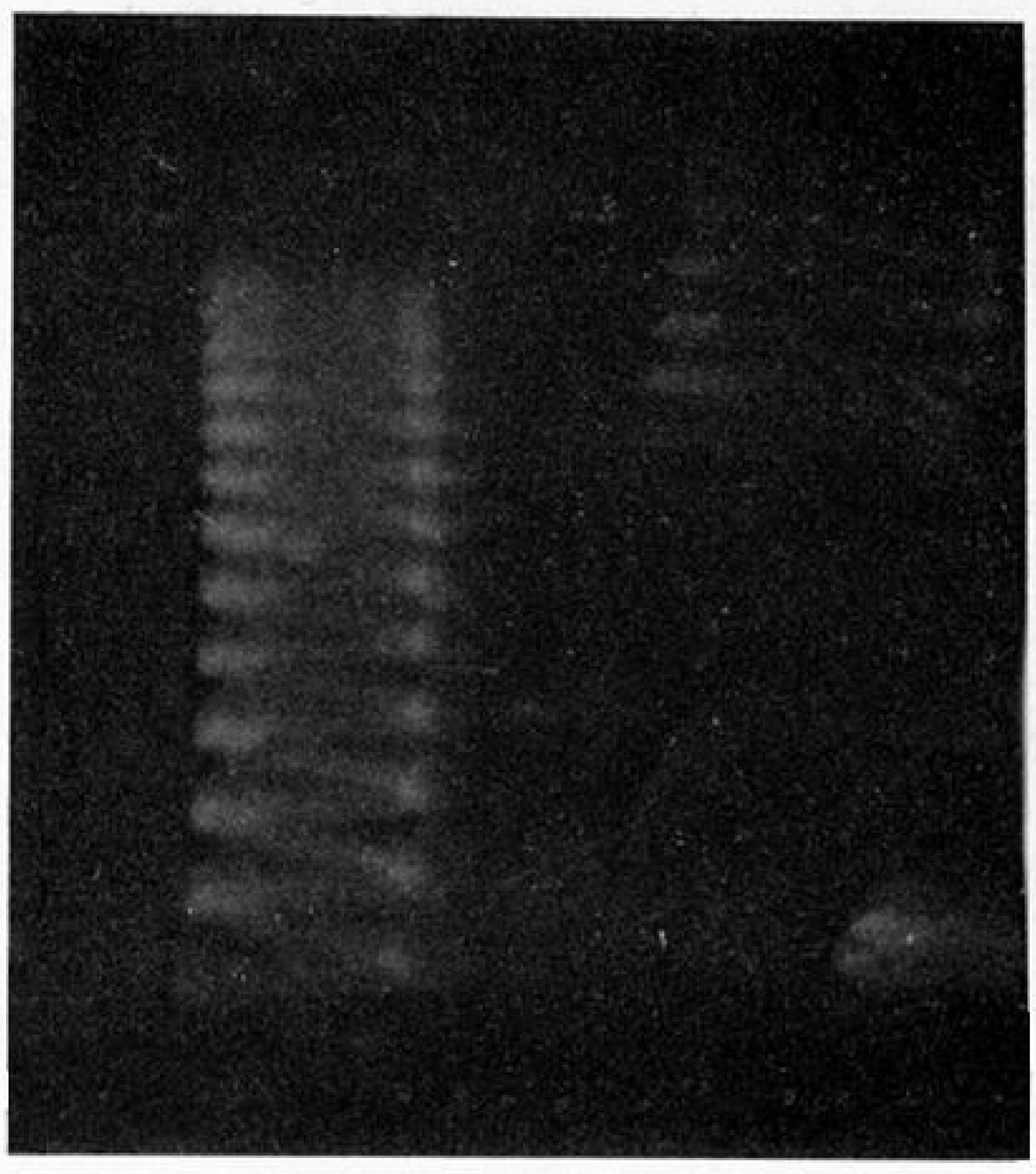


FIG. 41.



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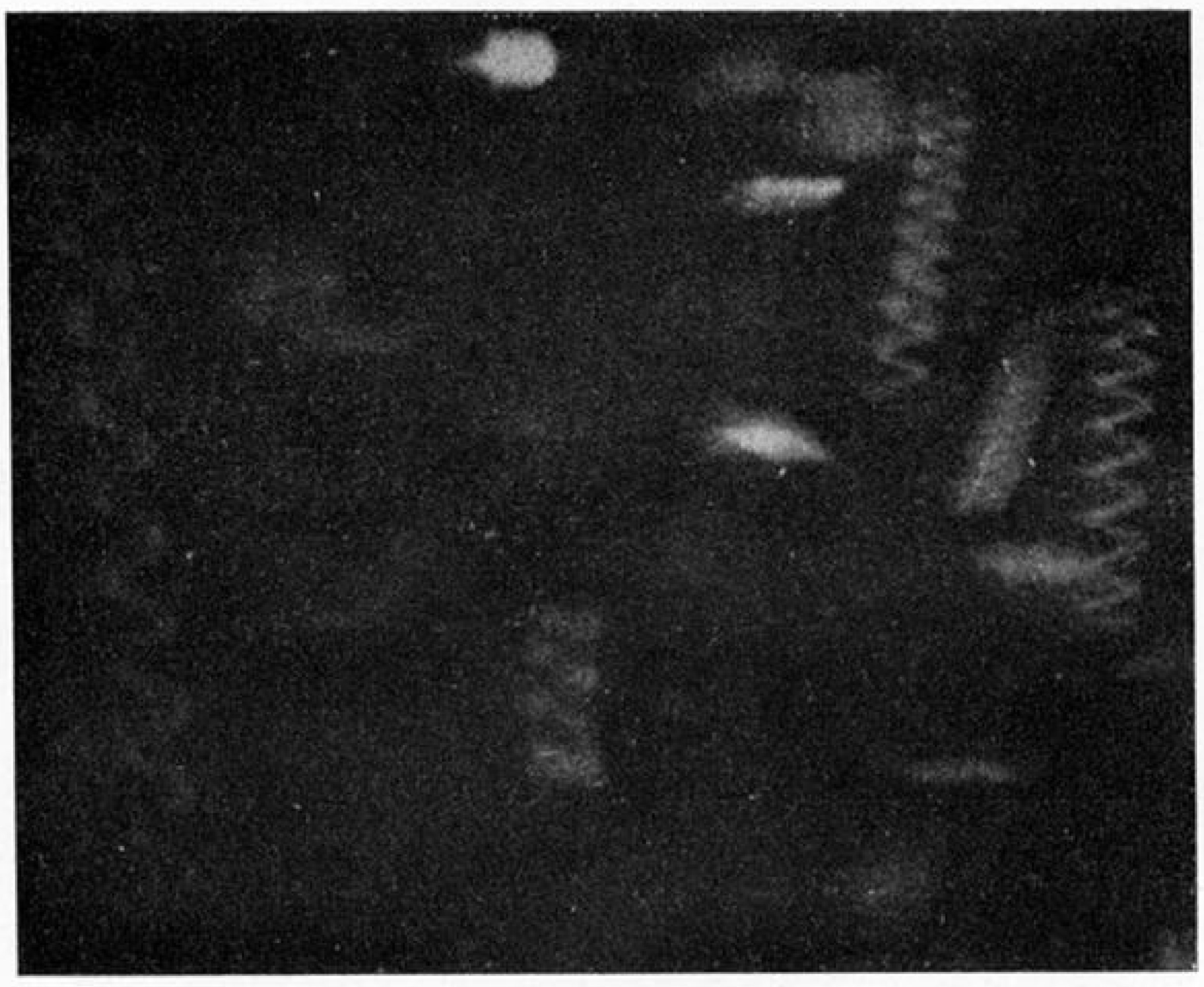

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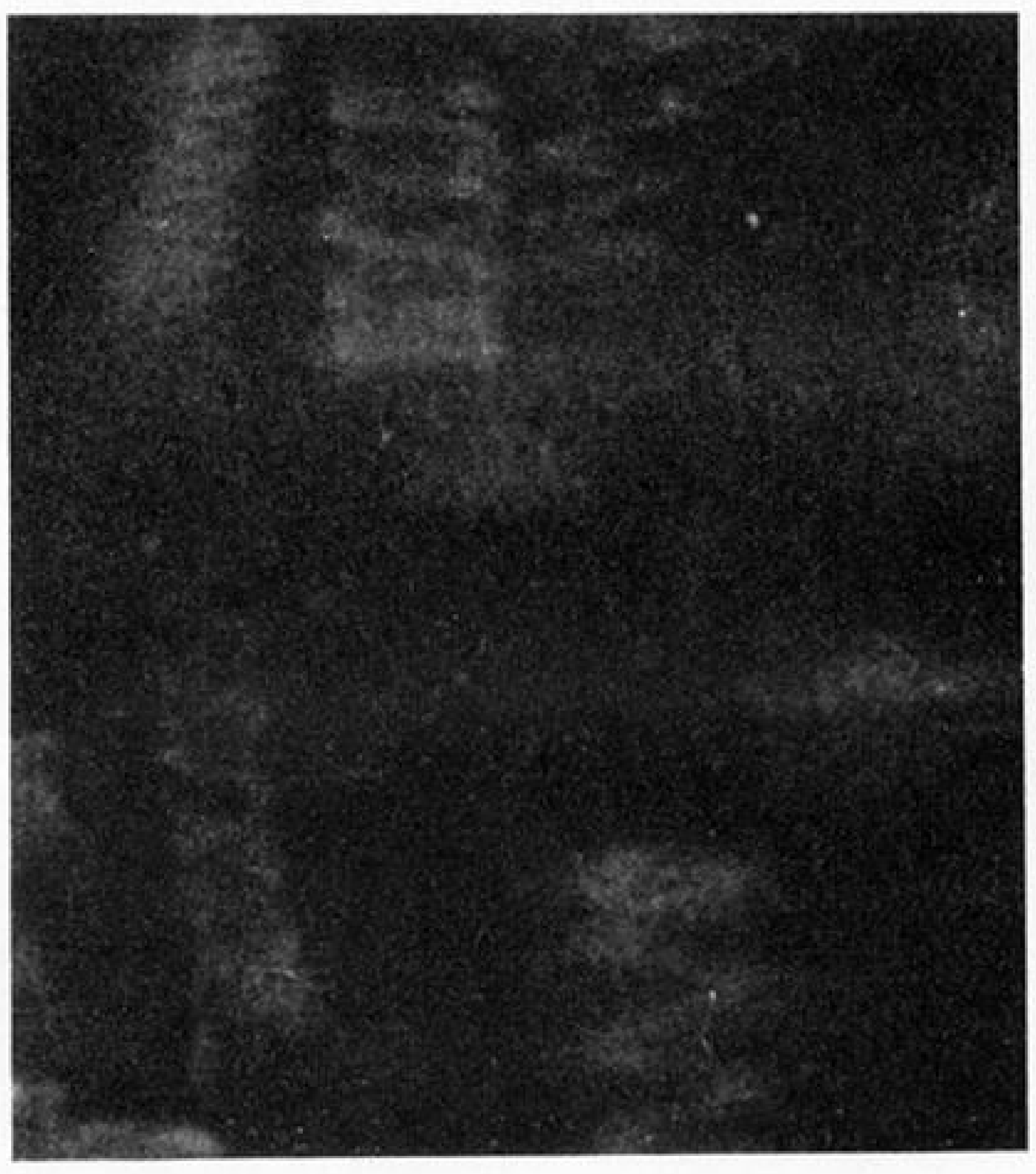

FIG. 44.

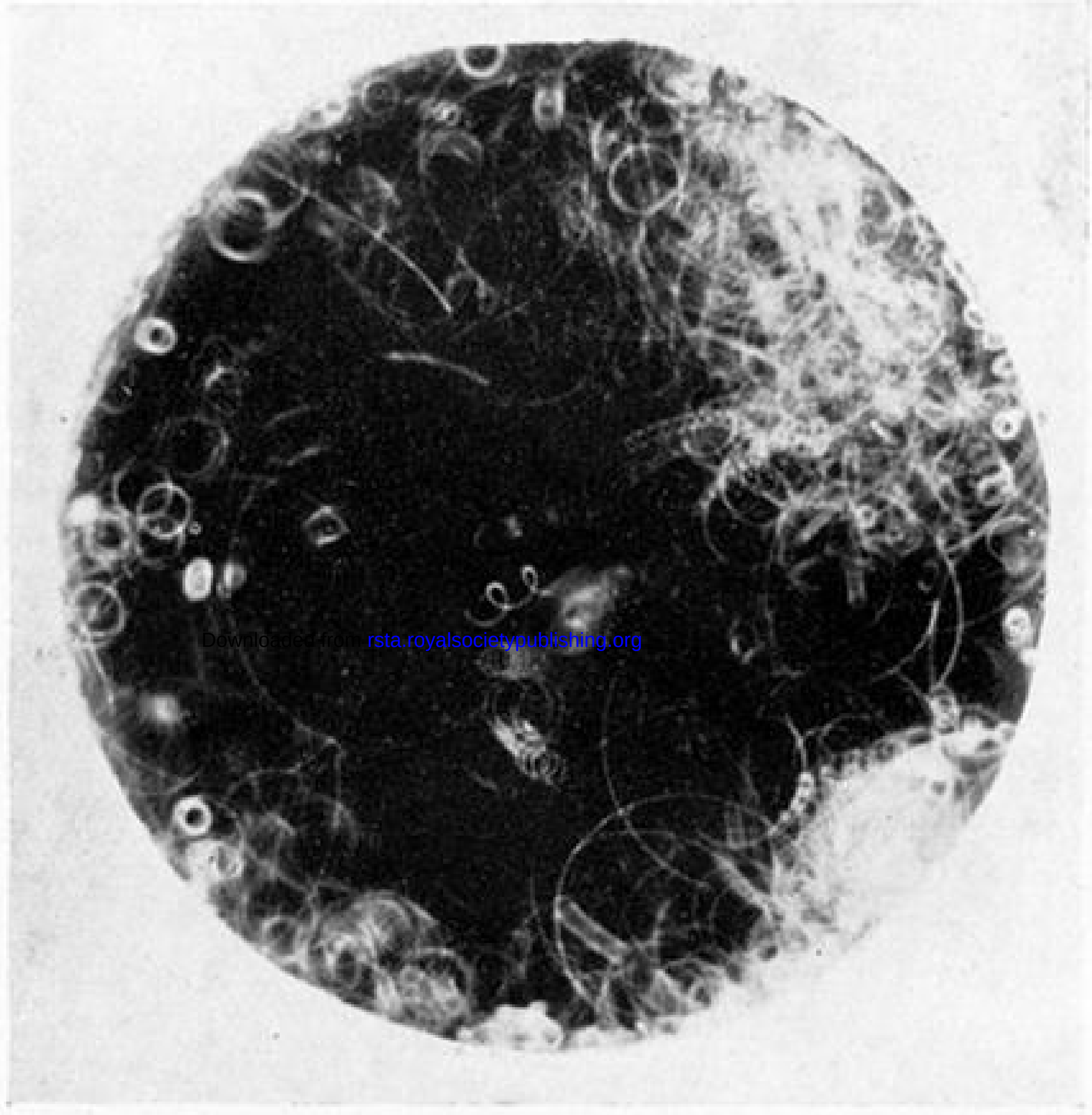

FIG. 45.

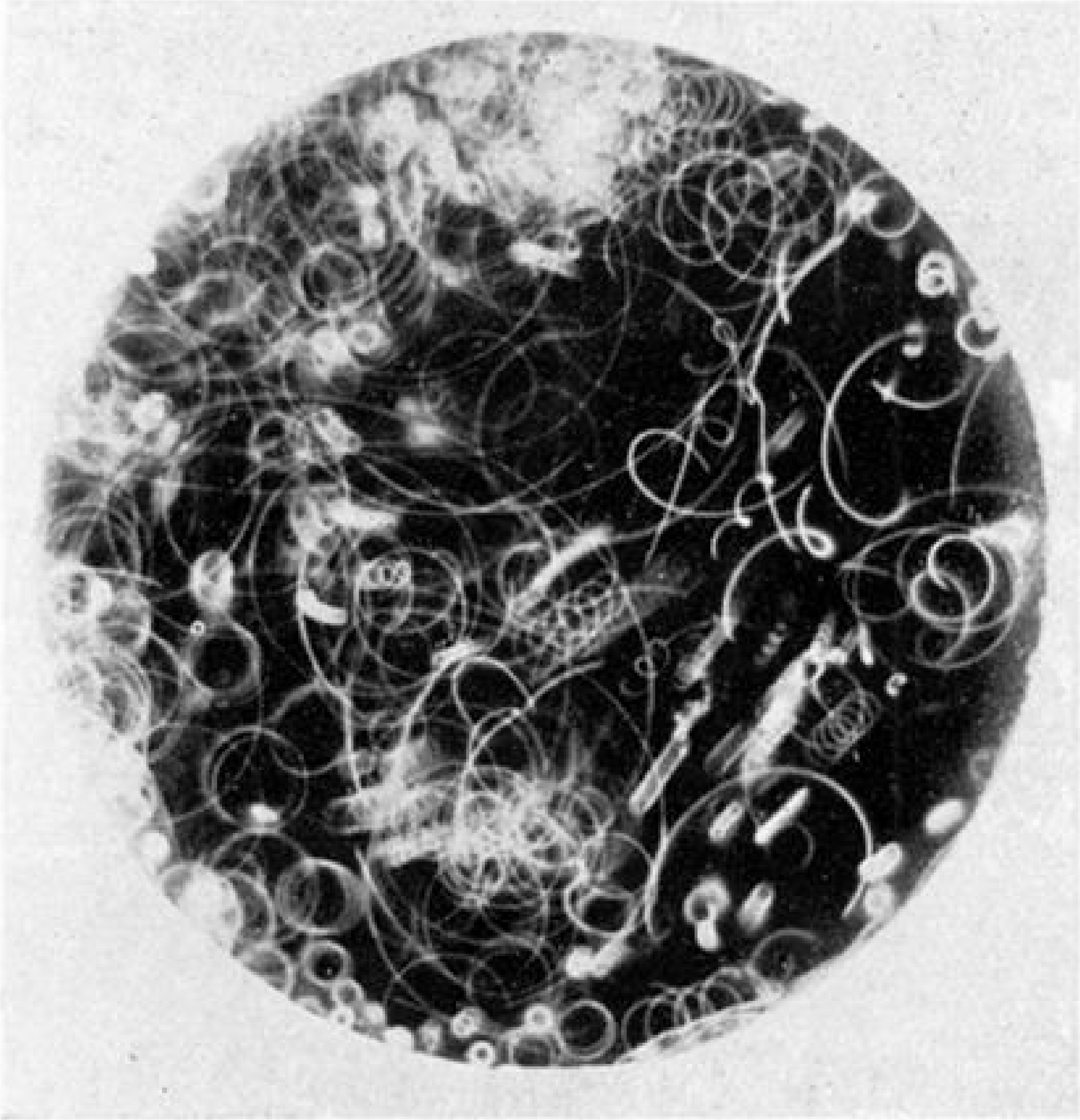

FIG. 46.

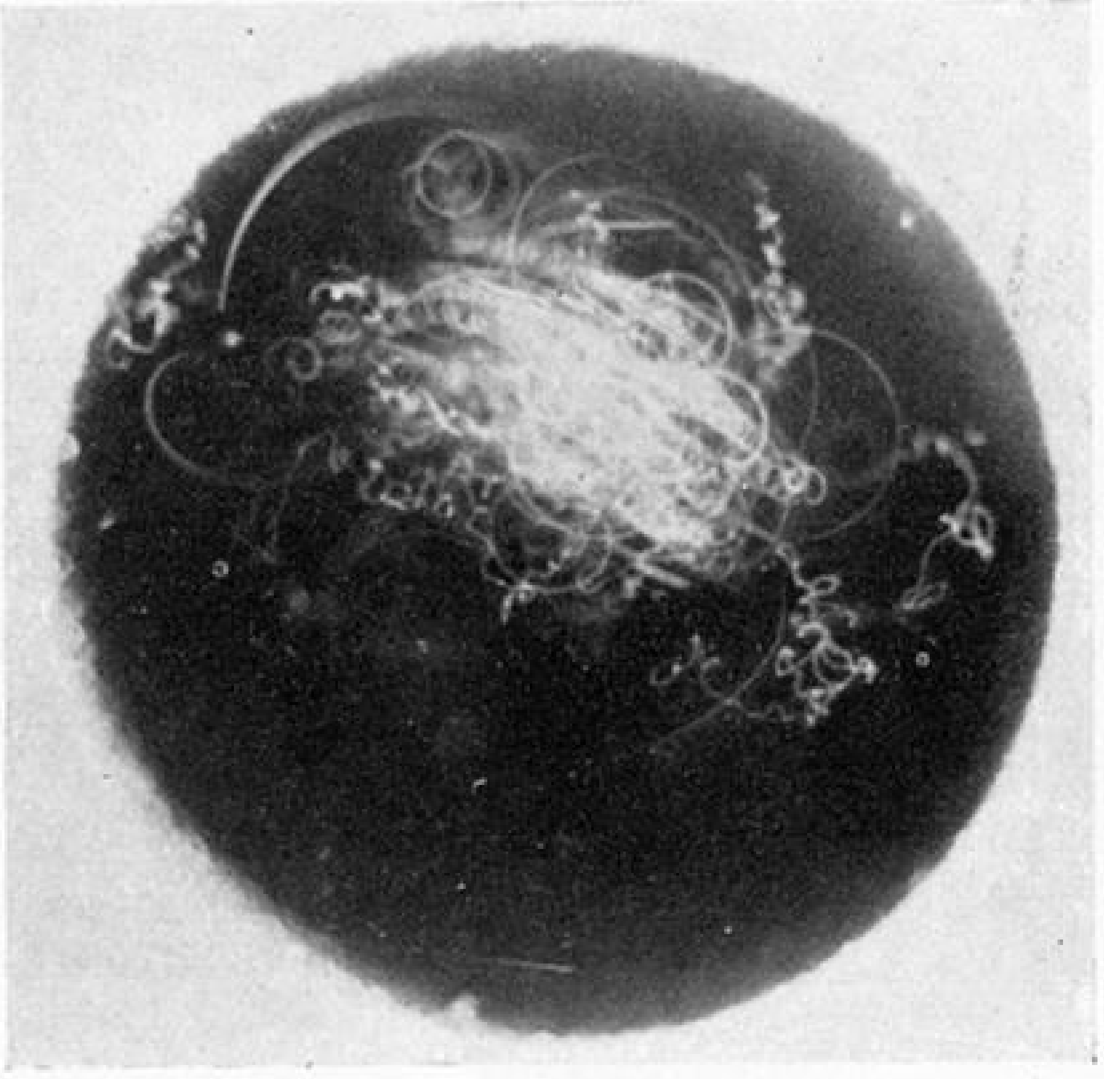

FIG. 47.

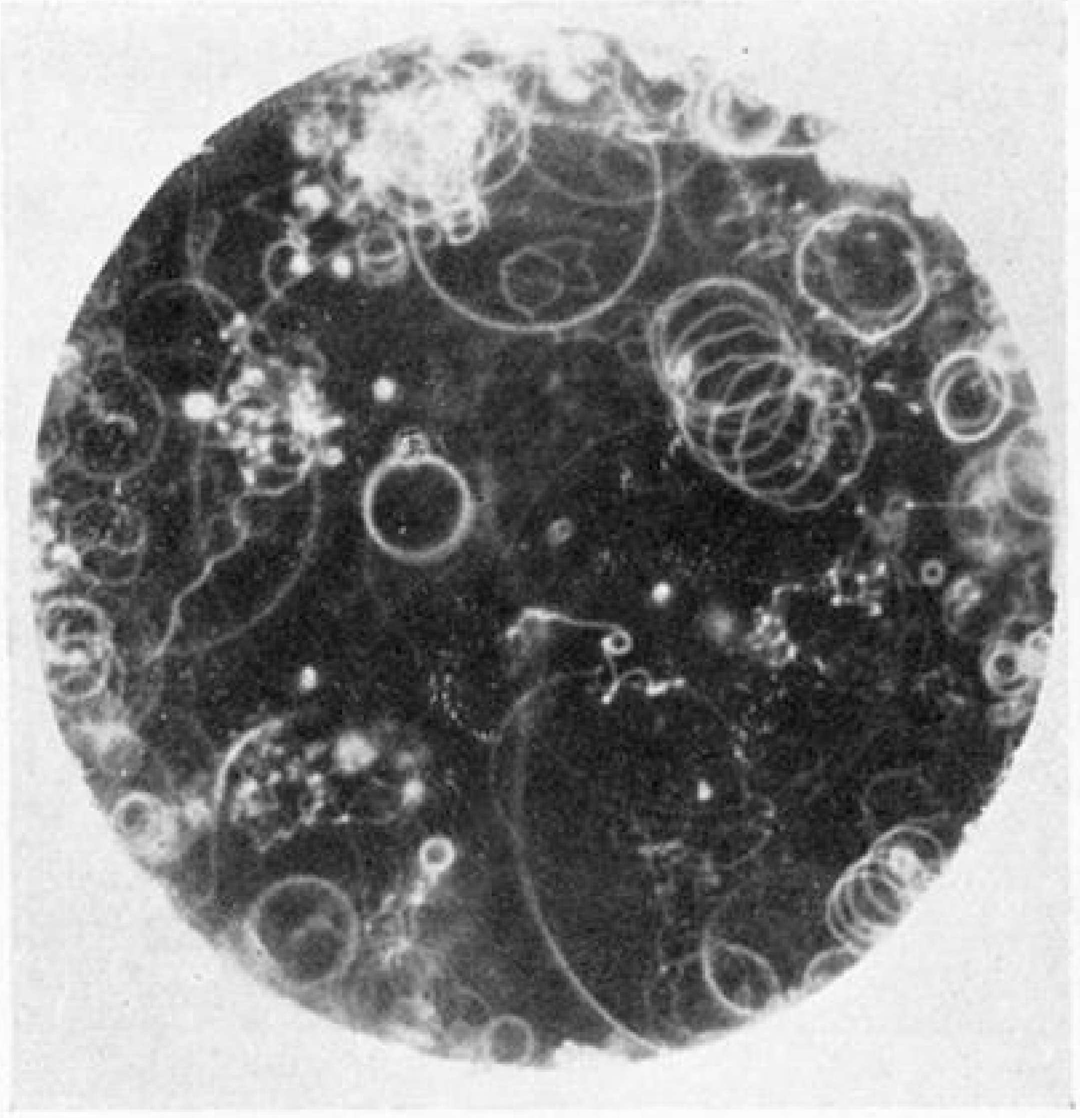

FIG. 48.